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Faculty of Earth and Universe Sciences

Department of Earth Sciences

Fluid mechanics

Summary of Course and Problem session

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Foreword

Foreword

Fluid mechanics holds a fundamental place in scientific and technical education, due to its numerous applications in physics, engineering, environmental sciences and civil engineering.

This course has been specifically prepared for first-year university students, as well as for anyone seeking to acquire the essential concepts of fluid mechanics, regardless of their background or field of study. It covers the basic principles from fluid statics and dynamics to the understanding of fundamental equations, with concrete examples drawn from everyday phenomena and industrial applications.

Throughout the document, priority is given to clarity of explanation and pedagogical progression, in order to facilitate understanding and assimilation of the concepts. This material provides:

- *an introduction to the main concepts and physical reasoning related to fluids.*
- *practical application through exercises and illustrative examples*.
- *quick access to definitions and tools necessary to continue training or to start projects involving fluid mechanics.*

The writing is based on reference works as well as on the teaching experience of several instructors. Any suggestions to improve this course material will be warmly welcomed, in the spirit of open and interactive learning.

Wishing you an enjoyable reading and every success.

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References

Introduction

Introduction

Fluid mechanics is a branch of mechanics that studies the behavior of fluids, liquids and gases, as well as their interactions with solids and their environment.

This manuscript is intended primarily for students from various fields and specializations in which fluid mechanics represents a fundamental and useful discipline.

Chapter I is devoted to the properties of fluids, such as density, specific weight, and viscosity, among others. It presents the fundamental law of fluid statics and analyzes the forces exerted by fluids on the walls of solid containers.

Chapter II focuses on Archimede's, principle. It examines the forces exerted by a fluid on a body, partially or fully immersed in it. Archimede's law is presented and demonstrated to highlight the relationship between the volume of displaced fluid and the resulting buoyant force. This chapter also discusses the equilibrium conditions of floating or submerged bodies.

Chapter III is dedicated to the study of the flow of ideal fluids. It introduces and demonstrates the fundamental equations governing such flow, including the continuity equation and Bernoulli's equation. These relations form the basis for numerous applications in hydraulics.

Chapter IV addresses the flow of real fluids. It introduces the concept of flow regimes, laminar and turbulent, and explains the mechanisms associated to internal friction forces that arise in a viscous fluid in motion. This chapter also details the methods for calculating head losses caused by these frictions, both in pipelines and in hydraulic systems. These concepts are essential for understanding the behavior of real systems and for designing hydraulic and civil engineering installations.

At the end of each chapter, a set of exercises, with and without solutions, is provided. This allows students to test their knowledge and understanding. It alsoà allows them to prepare for examinations.

Chapter 1: Fluid Static (Hydrostatic)

Chapter 1: Fluid Static (Hydrostatic)

1. Definition

Hydrostatics is the study of fluids at rest or in static equilibrium. Under these conditions, there is no distinction between ideal fluids and real fluids. When at rest, viscosity does not come into play. The static of fluids is based on the fundamental equation of static and Pascal's principle.

2. Characteristics of a fluid

A fluid is a deformable material composed of molecules that can move relative to each other. Fluids include liquids, gases, plasmas, and certain types of solids. Liquids have a defined volume (determined by the shape of their container) and exhibit free surfaces, and they are generally incompressible. Gases, on the other hand, expand to fill any container and are compressible. Fluids are characterized by properties that allow them to be described in a particular state. These properties include:

2.1 The density of mass

The density (ρ) of a substance is the amount of matter per unit volume, or the ratio of the mass (m) of the substance to the volume (V) it occupies. $\rho = m / V$. *Sa dimension est :* $[\rho] = M.L^{-3}$. *its unit is :* $Kg.m^{-3}$. *The specific masses of liquids, which are greater than those of gases, are generally less important than those of solids. Here are some examples :*

Liquid	ρ ($Kg.m^{-3}$)	*Gas*	ρ ($Kg.m^{-3}$)
Ethanol	791	*Hydrogen*	*0.085*
Olive oil	908	*Methane*	*0.717*
water	998	*Ammonia*	*0.718*
Sea water	1 030	*Nitrogen*	*1.163*
Glycerol	1260	*Dry air*	*1.205*
Mercury	*13546*	*Oxygen*	*1.330*

Density of a few liquids and gases, at 20 ° C. and a pressure of 1 atm.

2.2 The density

The density (d) does not appear explicitly in the fundamental laws of mechanics. However, this practical quantity allows for the classification of fluids relative to a reference. It is defined as the ratio of the density (ρ) of the fluid to the density (ρ_0*) of a reference substance.:* $d = \rho / \rho_0$. *Among the reference compounds, we have :*

Pure water for liquids: ρ_0 = 1000 Kg.m^{-3}, à 04 C° et 1 atm (1,013 10^5 Pa)

ρ_0^{15} = 999.1 Kg.m^{-3}, à 15 C° et 1 atm

Dry air for gas: Under the same temperature and pressure conditions as the fluid to be evaluated : ρ_0 = 1.205 Kg.m^{-3}, à 20 C° et 1 atm.

ρ_0^{15} = 1.293 Kg.m^{-3}, à 00 C° et 1 atm

In the case of an ideal gas: $d = \frac{\rho_{gaz}}{\rho_{air}} = \frac{M_{gaz}.P}{R.T} . \frac{R.T}{M_{air}.P} = \frac{M_{gaz}}{M_{air}}$

Under the same temperature and pressure conditions, the molar mass of the air (M_{air}) is from 29 g/mol. We deduce: M_{air} =29 d.

2.3 The compressibility

Compressibility distinguishes liquids from gases. A liquid occupies a fixed volume, which changes only minimally under significant variations in pressure or temperature. In contrast, a gas always expands to fill the volume available to it, making it a compressible or expandable fluid. Compressibility reflects a fluid's ability to undergo changes in volume due to an increase in pressure at constant temperature. The density of an incompressible fluid, such as a liquid, remains constant throughout. However, the density of a compressible fluid, like a gas, can vary from one point to another. An increase in pressure leads to a decrease in volume, and vice versa. To quantify this effect, the isothermal compressibility coefficient is used, typically expressed with a negative sign.

$$C = (- 1 / V) (\delta V / \delta P)_T$$

From the dimensional point of view, the compressibility coefficient is the inverse of the pressure, i.e. (M^{-1} L T^2). It is expressed in Pa^{-1} in the system (SI).

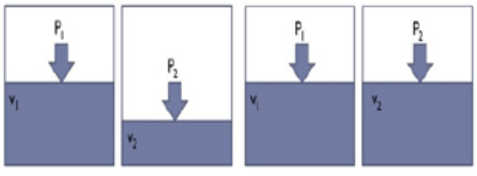


Diagram illustrating the notion of compressibility.

$\chi_{gaz} >> \chi_{liquide}$

Liquid and gas	*Compressibility (Pa^{-1})*
Water	*04.58 10^{-10}*
Benzene	*09.44 10^{-10}*
Methanol	*12.11 10^{-10}*
Air	*00.00 10^{-05}*

Orders of magnitude of the coefficient compressibilities of a few liquids and gases

For ideal gases, we deduce from the equation of state: χ = 1/P.

2.4 The viscosity

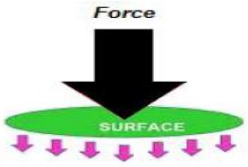
The agitation of molecules leads to a microscopic transfer of momentum from one particle to its neighboring particles when their velocities differ. This transfer gives rise to viscosity. Viscosity is defined as the resistance of a fluid to deformation and its flow or movement. Fluids with low viscosity flow more easily than those with high viscosity. Every real fluid has some viscosity. In an ideal fluid, however, no frictional forces oppose the movement of fluid particles relative to each other.

if $\eta = 0$, the fluid is said to be perfect or ideal, it flows without friction.
Si $\eta \neq 0$, the fluid is said to be real, it flows with friction.

Illustration of the notion of viscosity.

2.5 The pressure

The pressure (P) at a point M, is the quotient of the force by the surface. It is a scalar quantity. P = F / S. It is expressed in Pascals (Pa). Its dimension is: $M.L^{-1}.T^{-2}$



There are other units of pressure :

1 bar = 10^5 Pa
1 torr = 1.334 Pa
1 barie = 10^{-1} Pa
1 mmHg= 133.3 Pa

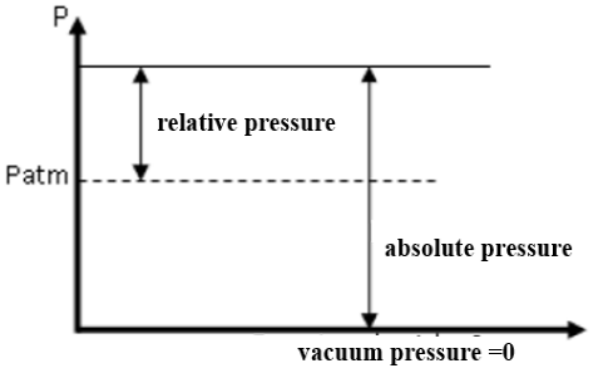
1 atmospheric pressure = 1.01325 10^5 Pa

The pressure has different types :

The Absolute Pressure : *Denoted (P_0), it is the pressure measured with respect to the absolute zero of pressure (total absence of pressure or perfect vacuum).*

The atmospheric pressure : *Also known as barometric pressure, it is the pressure exerted by the earth's atmosphere. The normal at sea level is 1.013 bar (760 mmHg). This pressure decreases with altitude.*

The effective or relative pressure (Pr) : *This is the pressure measured relative to atmospheric pressure. It can be positive if we work under pressure, zero if we work under atmospheric pressure and negative if we work under reduced pressure. The latter is connected to the absolute pressure by:* $\boldsymbol{P_a = P_r + P_{atm}}$.



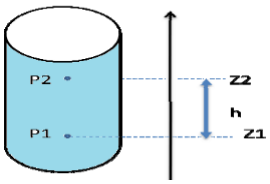
Representative diagram of the different types of pressure.

3. Fundamental Principle of hydrostatics

3.1 Case of incompressible fluids

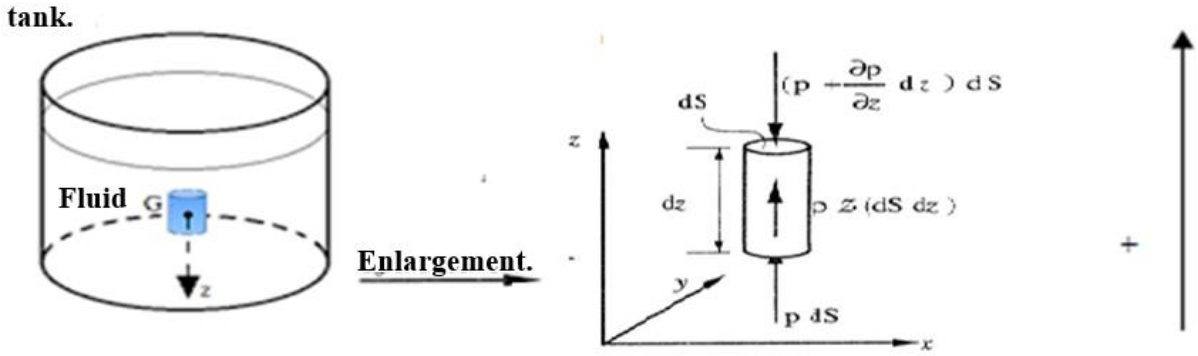
Consider a liquid with constant density, in equilibrium, and subjected to gravity. Let an elementary volume dV be defined by a cylinder with cross-sectional area dS and height dz (as shown in the diagram below).

$\int_{P_1}^{P_2} dP = \int_{Z_1}^{Z_2} -\rho.g.dz$ $\quad P_2 - P_1 = -\rho \,.\, g \,.\, (Z_2 - Z_1)$



Either : $P_1 = P_2 + \rho \,.\, g \,.\, (Z_2 - Z_1) = \rho \,.\, g \,.\, h$

The pressure increases linearly with depth.



The forces that act on the volume element dV = dS dz, along the vertical are:

- *The forces of volume or weight:* $\vec{P} = -\rho.g.dz.dS\,\vec{k}$
- *Surface forces or pressures: in Z :* $P(x,y,z).dS.\vec{k} = p.dS.\vec{k}$

  en z+dz : $-P(x,y,z+dz).dS.\vec{k}$

The formula for the finite increments gives:

$$P(x,y,z+dz).dS.\vec{k} = -\left(P(x,y,z) + \frac{\partial p(x,y,z)}{\partial z}\, dz.\right).dS.\vec{k}$$

The conditions of equilibrium of forces according to z are :

$$P(x,y,z)\,dS - \left(P(x,y,z) + \frac{\partial p(x,y,z)}{\partial z}.dz\right).dS - \rho.g.dz.dS = 0$$

since $P(x,y,z) = P$ *,we can write:* $P\,dS - P.dS - \frac{\partial P}{\partial z}.dz.dS - \rho.g.dz.dS = 0$

$-\frac{\partial P}{\partial z} - \rho.g = 0 \quad \Rightarrow -\frac{\partial P}{\partial z} = \rho.g$, *more generally:*

$$\overrightarrow{grad}\, p = -\rho\vec{g} \;\; ou \;\; \nabla p = -\rho\vec{g}$$

The first term represents the pressure forces per unit volume, the second the volume forces per unit volume. In hydrostatics, we generally only consider the Earth's gravitational field. The previous equation is called the fundamental equation of hydrostatics. The integration of this equation between Z_1 *and* Z_2 *is written :*

Some special cases

Horizontal plane

On a horizontal plane the resultant of the forces is zero.

$$\sum F_x = 0 \Rightarrow P(x)dz\,dy - p(x+dx).dz.dy = 0$$

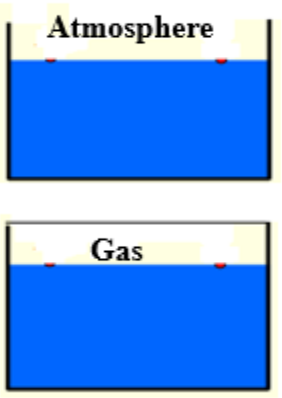
By dividing this equation by the product dx. dy.dz :

$$\frac{P\left(x+dx-p(x)\right)}{dx} = 0 \quad which\ amounts\ to:$$

$$dx \to 0$$

$$\frac{dP}{dx} = \lim_{dx \to 0} \frac{P(x+dx)-P(x)}{dx} = 0 \Rightarrow P = cst$$

On a horizontal plane, all the pressures are equal (Isobaric pressures.)



Whatever the position of a container, the free surface, in contact with the atmosphere or a gas, the surface is horizontal.

Immiscible liquids

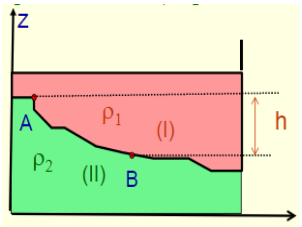
Let be two immiscible fluids with a density ρ_1 and ρ_2, in the same container. Let be two points A and B of the separation surface, assumed to be non-horizontal.

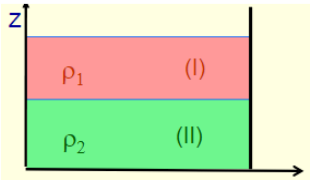
In the fluid 1 : P_A-$P_B = \rho_1 . g . h$

hence $\rho_1 . g . h = \rho_2 . g . h \Rightarrow (\rho_1 - \rho_2).h.g = 0$

In the fluid II : P_A-$P_B = \rho_2 . g . h$

Or $\rho_1 - \rho_2 \neq 0$ *et* $g \neq 0$ *so* $h = 0$.

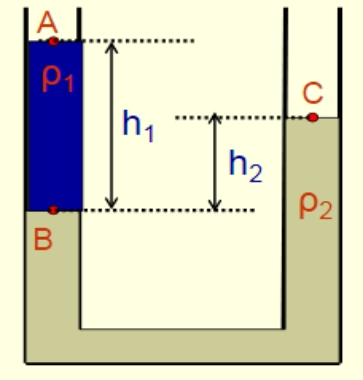




The separation surface of two immiscible liquids at rest is horizontal .

Communicating vessels with several liquids

Let be a liquid with a density of mass ρ_1, in a plastic tube U.
We pour into the other branch a liquid with a density of mass ρ_2.



Let the points A, B and C be such that:

$P_B - P_A = \rho_1 . g . h_1$ *et* $P_B - P_C = \rho_2 . g . h_2$

Since : $P_A - P_C = P_{atm}$, $\rho_1 / \rho_2 = h_2 / h_1$

The differences in level of two immiscible liquids in communicating vessels are in inverse ratio to their density

Remarque

if $\rho_1 = \rho_2$ so $h_1 = h_2$

A fluid is at the same height in two communicating vessels

3.2 Case of compressible fluids

As the equation dp=−ρ⋅g⋅dzdp = -\rho \cdot g \cdot dzdp=−ρ⋅g⋅dz indicates, pressure variations depend on altitude. The negative sign signifies that pressure decreases with increasing altitude. If we assume that the density of a liquid is constant and independent of temperature or pressure, this is not the case for gases. The variation in temperature is given

$$by : \rho = \rho_0 . g . \frac{P}{P_0} . \frac{T_0}{T}$$

The variation of the pressure, for an ideal gas, is : $\frac{dP}{dz} = -\rho_0 . g . \frac{P}{P_0} . \frac{T_0}{T}$. *At constant temperature (isothermal conditions $T = T_0$), dP/dz becomes :*

$$\frac{dP}{P} = -g . \frac{\rho_0}{\rho} . dz .$$

The density of the compressible fluid, if it can be assimilated to an ideal gas, is given by $\rho_0 = \frac{M.P_0}{R.T}$.

$\int_{P_0}^{P} \frac{dP}{P} = -\frac{\rho_0}{P_0} . g \int_{z_0}^{z} dz$. *Either* $P = p_0 . exp(-\frac{\rho_0}{P_0} . g(z - z_0) = p_0 . exp(-\frac{M.g}{R.T}(z - z_0)$

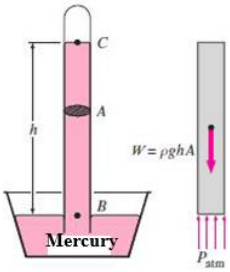
This equation is known as the barometric equation and shows that the pressure decreases exponentially when with altitude.

4 Pressure measurements

Pressure can be measured using various devices and instruments, including:

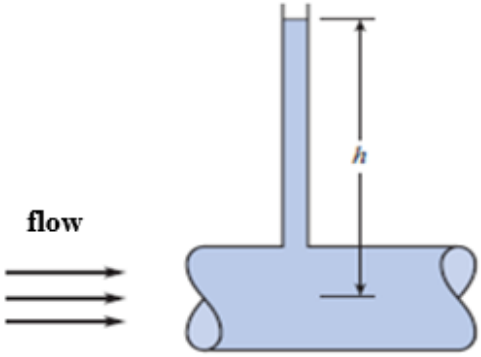
4.1 The barometer

It is an instrument for calculating the value of atmospheric pressure. The Italian Evangelista Torricelli (1608-1647), is the first to have measured the atmospheric pressure by a simple experiment.



4.2 The piezometer

It is the simplest pressure measuring instrument. It is a transparent tube, vertical or inclined, connected to the point where we want to determine the pressure. This being none other than the height of water which rises in the tube.



4.3 The manometer

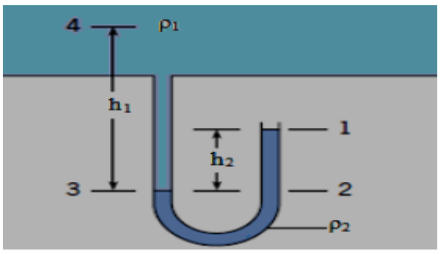
There are several types of manometers, to measure the pressure of a fluid at rest or in motion. The single column manometer, the U-tube manometer, The inclined tube manometer, the bell manometer, the differential manometer... etc.

Standard

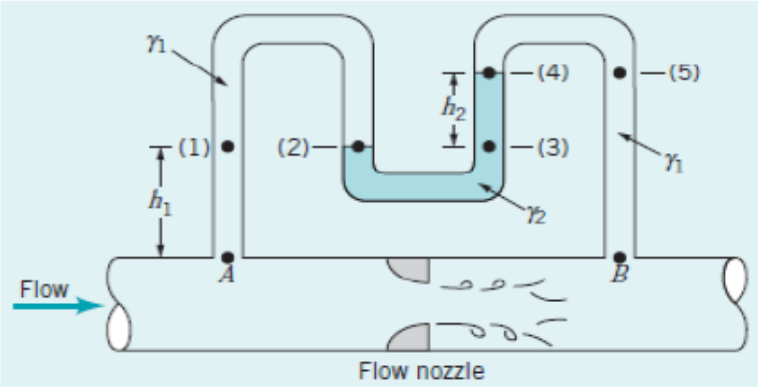
Electronics

U-shaped pressure gauge and differential pressure gauge.

The U-shaped manometer consists of a U-shaped tube, one end of which is connected to the measuring point and the other to the open air. The tube contains either mercury or a liquid denser than the fluid whose pressure is to be measured.



The differential manometer is a tube connected between two points, where we want to determine the pressure difference or the piezometric height. It can be a single liquid with an air inlet valve, or two liquids

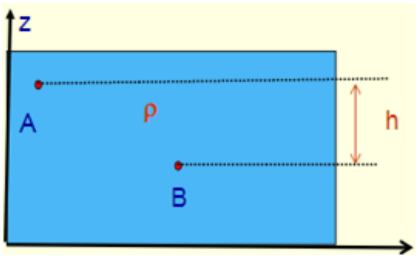


4 Pascal's principle and theorem: Application

(Blaise Pascal French mathematician and philosopher, 1647)

Theorem

Any variation in pressure at one point of a liquid causes the same variation at all its points. The liquids transmit the pressure in full. If at a point in a fluid the pressure increases by ΔP, all points undergo the same increase in pressure.



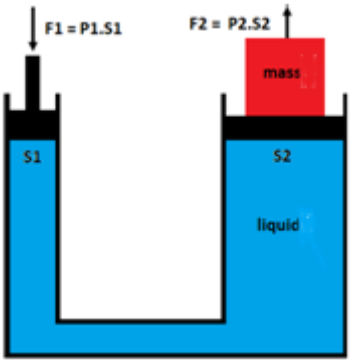
If the A-pressure increases by ΔP

The pressure at B also increases by

$$\Delta P = P_A - P_B = Cst$$

Application: The hydraulic

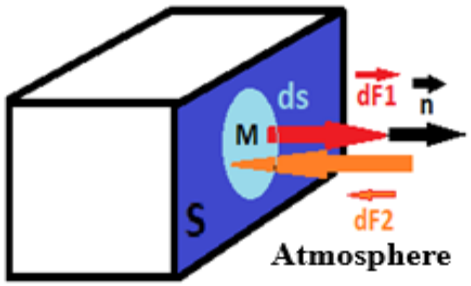
A very interesting application of Pascal's principle is the hydraulic press. ***Principle****: A force* $F_1 = P_1 \cdot S_1$ *is applied to surface* S_1*. Considering surface* S_2 *of a piston at the outlet, which is larger than the inlet surface* S_1*, a significant force* $F_2 = P_2 \cdot S_2$ *will be generated on surface* S_2*.*



The liquid transmits the increase in pressure it undergoes in all directions and uniformly. If $P_1 = P_2$*, then* $F_2 = F_1 (S_2 / S_1)$*. Since* $S_1 < S_2$*, it follows that* $F_2 > F_1$*. The press multiplies the force applied to surface* S_1*. Therefore, it is possible to lift loads placed on surface* S_2 *by applying a relatively smaller force* F_1 *on* S_1*.*

5. Pressure forces

Let point M be a point in a fluid under static pressure P, surrounded by an elementary surface **ds** *with an outward normal* **n**.
The elementary force **dF** *applied to the surface element* **ds** *is the resultant of the elementary forces* dF_1 *and* dF_2*.*



$$d\vec{F} = d\vec{F_1} + d\vec{F_2} = \text{P.ds.}\,\vec{n} + P_{atm}.ds\,(-\vec{n}) = (P - P_{atm}).ds\,\vec{n} = P_{eff}.ds\,\vec{n}$$

If the surface is flat, the normal vector $\vec{n}$ *has a constant direction:* $F = \int P_{eff}\,.ds$

6. Pressure forces acting on the walls

6.1 Force of pressure acting on a horizontal wall

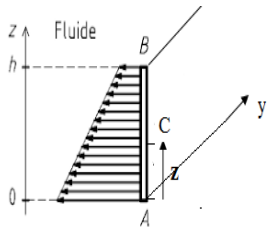
Consider a reservoir with a base surface SSS, open to the atmosphere, at a height h, containing a liquid with density of mass ρ. Since the surface is horizontal, the pressure is uniform across the

entire surface.

$$P_{eff} = \rho.g.h \quad \text{et} \quad F = \int P_{eff}\,.ds \Rightarrow F = \int \rho g.h\,.ds = \rho g.h \int ds = \rho.g.h.S$$

6.2 Pressure exerted on a vertical wall: Point of application of the force

The pressure at any point (C) on the surface is given by:



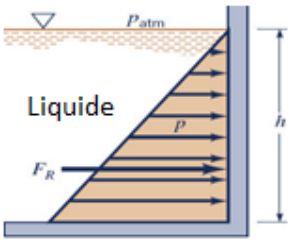
$$P_{C_{eff}} = \rho.g.(h-z) \Rightarrow F = \int P_{eff}\,.ds$$

$$F = \int \rho.g.(h-z)\,.ds \quad \textit{where} \quad ds = dy.dz$$

$$F = \iint \rho.g.(h-z)\,dy.dz = \int_0^L dy\,.\int_0^h \rho.g.(h-z)\,dz$$

$$F = \rho.g.L\int_0^h .h\,dz - zdz = \rho.g.L\left(h^2 - \frac{h^2}{2}\right)$$

$$F = \rho.g.L\,\frac{h^2}{2}$$



Graphical representation of pressure on a vertical wall

Center of pressure

This is the point of application (C) of the force F, or the resulting pressure. Let Δ be the axis around which the wall rotates under the effect of the pressure forces. Therefore, we have : $\mathcal{M}\left(\vec{F}\right)_{/\Delta} = \mathcal{M}\,(\int \mathrm{d}\vec{\mathrm{F}}\,)_{/\Delta}$

$\mathcal{M}\left(\vec{F}\right)_{/\Delta}$ *representing the moment of force* $\vec{F}$ *with respect to the axis (Δ) It is expressed as the product OC.F.*

$$\mathcal{M}\left(\vec{F}\right)_{/\Delta} = \mathrm{OC}\,.\,\mathrm{F} = \mathrm{OC}.\rho.g.L.\frac{h^2}{2} = \mathrm{z}.\mathrm{dF} \qquad (1)$$

$$\mathcal{M}\,(\int \mathrm{d}\vec{\mathrm{F}}\,)_{/\Delta} = \int \mathrm{z}.\mathrm{dF} = \iint z.\rho.g.z.dy.dz = \rho.g.\int_0^{\mathrm{L}} \mathrm{dy}.\int_0^{\mathrm{h}} \mathrm{z}^2\ \mathrm{dz}$$

$$\mathcal{M}\,(\int \mathrm{d}\vec{\mathrm{F}}\,)_{/\Delta} = \rho.g.L.\frac{h^3}{3} \qquad (2)$$

By equating (1) et (2), We obtain: OC = 2.h/3

If we consider a surface of any geometry immersed in a liquid, three questions arise:

1- *What is the magnitude (intensity)of the force acting on the surface?*
2- *Where is the point of application (C) of this force located?*
3- *What is the direction of this force?*

6.3 Inclined flat wall

Consider a wall with surface area SS and center of gravity C, immersed in a liquid and inclined at an angle with respect to the horizontal. Let dS represent the surface element. The pressure force on dS can be determined.

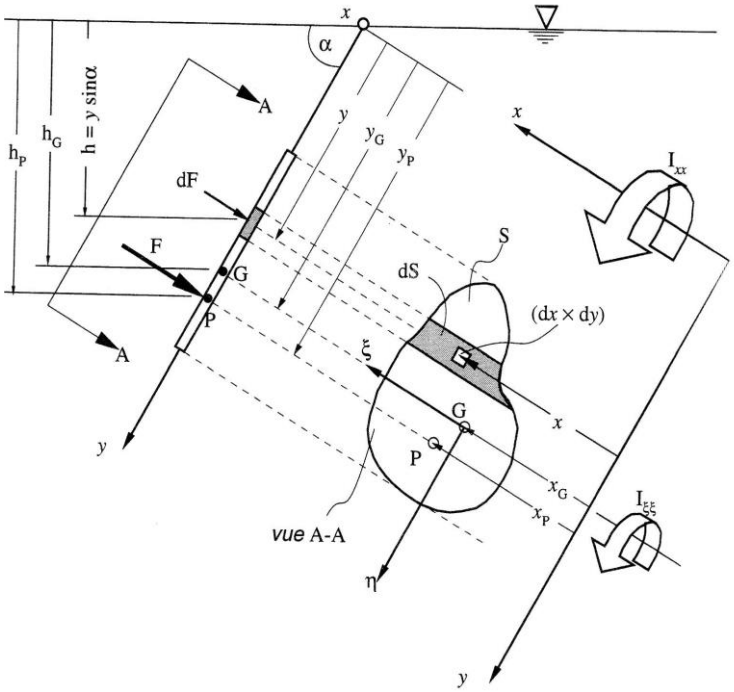
The pressure at point M M(x,y) is : $P_M = \rho\, g\, h$ avec : $h = y.\sin\theta$.

If atmospheric pressure is neglected, the force acting on dS is $dF = P.dS = \rho.g.y.sin\theta.dS$

The magnitude of the resultant force acting on the wall S is:

$$F = \int_S dF$$

$$= \int_S \rho.g.y.sin\theta.dS$$



$F = \rho.g.sin\theta\ y_G.S = \rho.g.h_G.S$.

where G is the center of gravity of the submerged surface, with coordinates: x_G , y_G

$$F = P_G.S$$

The magnitude (intensity) of the pressure force on surface S is equal to the pressure at the center of gravity of the surface multiplied by the area of the surface.

Note

The center of gravity of a two-dimensional plate is given by:

$$\overrightarrow{OG} = \frac{1}{p}\int_S p_i.\overrightarrow{OG_i}$$

p is the total weight of the plate, composed of subsets of weights pip_ipi and their corresponding centers of gravity GiG_iGi. If the plate is homogeneous (with weight evenly distributed), we have:

$S.x_G = \int_S x.dS$ *et* $S.y_G = \int_S y.dS$ *avec* $S.x_G = \int_S x.dS = \int_x \int_y x\,.dx.dy$

If x is independent of y, the double integral becomes :

$$S.x_G = \int_S x.dS = \int_x \int_y x\,.dx.dy = \int_x x.dx.\int_y dy$$

Center of pressure (or thrust)

The point of application of the force FFF is called the center of pressure (or thrust), denoted as G=(xp,yp). To determine the coordinates of this point, we will use the moments of force relative to the x- and y-axes:

$x_p.\int_S dF = \int_S x.dF$ *ou bien* $y_p \int_S dF = \int_S y.dF$

To find the ordinate of the center of pressure (or thrust) P, we will use : $dF = p.\,d\,S = \rho.g.\,h.\,dS$. *Substituting into the equation for y, we obtain:* $y_p \int_S dF = \int_S y.\rho.g.h.dS$ *, since* $h = y.\sin\alpha$, *The term on the right-hand side becomes:* $\int_S y.dF = \int_S y.\,\rho.g.y.\sin\alpha.dS = \rho.g.\sin\alpha.\int_S y^2.dS$

The term on the left is: $y_p \int_S \rho.g.h.dS = y_p \int_S \rho.g.y.\sin\theta.dS = \rho.g.y_p.\sin\theta.y_G.S$

We obtain: $y_p.y_G.S = \int_S y^2.dS$. *The ordinate of the center of pressure (or thrust) (*y_p*) will be expressed as:* $y_p. = \frac{\int_S y^2.dS}{y_G.S} = \frac{I_{xx}}{y_G.S}$

$I_{xx} = \int_S y^2.dS$ *is the second moment of inertia about the x-axis, passing through 0. For calculations, it is more convenient to replace Ixx with the moment of inertia I, relative to an axis parallel to it that passes through the center of gravity of the surface:*

$$I_{xx} = I_{xc} + S.{y_G}^2$$

As for the abscissa of the center of pressure (or thrust): P

By using the same approach as for y :

$\int_S x.dF = x_p.\int_S dF$ *avec* $dF = P.dS = \rho.g.h.dS = \rho.g.y.\sin\alpha\,.\,dS$

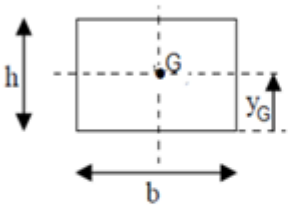
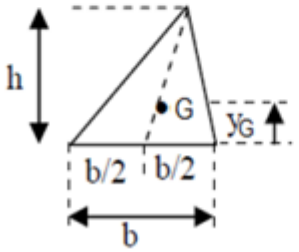
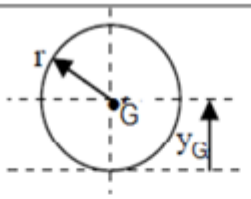
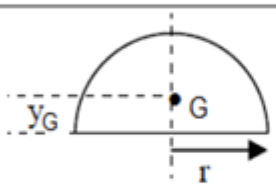
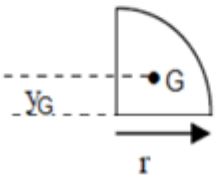
We obtain : $\int_S x.\rho.g.y.\sin\alpha.dS = x_p.\int_S \rho.g.y.\sin\alpha.dS$

D'où $\int_S x..y.dS = x_p.\int_S y.\,dS = x_p.y_G.S$

$$x_p = \frac{\int_S x.y.dS}{y_G.S} = \frac{I_{xy}}{y_G.S}$$

$I_{xy} = \int_S x.y.dS$ *it is the moment of inertia of surface S in the plane (0, x, y).The center of pressure (thrust) should not be confused with the center of gravity; it is always located below. Its position A on the plate does not change with α; A and G are fixed points.*

Centers of gravity and moments of inertia for common surfaces: Example

Type of surface	Geometric shape	Center of gravity	Surface	Moment of inertia I_{xG}
Rectangle		$\frac{h}{2}$	bh	$\frac{bh^3}{12}$
Triangle		$\frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle		r	πr^2	$\frac{\pi r^4}{4}$
Semicircle		$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4$
Quarter circle		$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$	$\left(\frac{\pi}{16}-\frac{4}{9\pi}\right)r^4$

Example: *Determine the hydrostatic force on the circular wall AB and its center of pressure. We provide ρ = 1000 kg/m^3 et g = 9.81 m/s^2.*

Solution: *The hydrostatic force* $F = \rho.g.h_G$ - $F = 1000 .9.81. (6-0.5).\pi.(0.5)^2 = 42.354$ *KN*

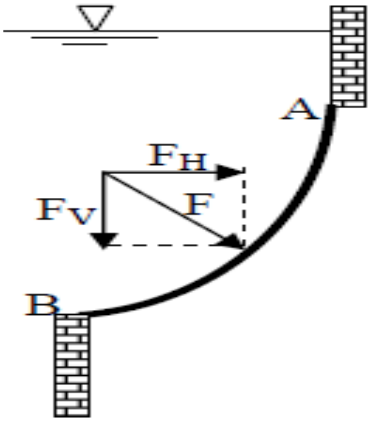
The center of pressure: $h_p = y_p = y_G + (I_{xG} / y_G.S)$ $y_G = h_G = 5.5$ *m*

et $I_{xG} = \pi D^2/64 = 0.049\ m^4$ $h_p = 5.511$ *m*

6.3 Curved wall

The resultant force FFF of the pressure exerted by a liquid on a surface is expressed as: $\vec{F} = \overrightarrow{F_H} + \overrightarrow{F_w}$

$\overrightarrow{F_H}$ *: The horizontal component, whose norm is equal to the force exerted by the fluid on the vertical projection of the surface. Its point of application is the center of pressure of the projected surface.* dF = p.dS ou $F = \int_S p.dS$



Integration is not always feasible because the force F changes direction on the wall. In such cases, the components are evaluated separately. (F_H) *et* (F_V).

(F_H) *is written:* $dF_H = dF . \sin\theta = p.dS.\sin\theta = p.dS_v$

(F_V) *is determined by:* $F_V = \int_S p.dS.\cos\theta.dS$

The norm is equal to the weight of the liquid directly above the surface. Additionally, it is important to note that dS cos(θ) represents the projection of the surface S onto a horizontal plane. Calculating the two components, F_H *and* F_V *, allows for the determination of the resultant force F, which is given by :* $F = \sqrt{F_H^2 + F_V^2}$

Example: *A radial valve is located at the base of a vertical wall. The width of the valve is L = 5m and its radius is R = 4m. Calculate the resulting force exerted on the valve.*

Solution:

Horizontal force: $F_H = \rho .g. h_G .Sx = \rho.g.h_G.R.L.$

Sx being the projection of the curved surface AB onto a vertical plane.: FH = 1000 9,81 5 20 = 981 KN

Application point: $h_p = y_p = y_G + (I_{xG} / y_G.S)$

$y_G = y_G = 5\ m$ *et* $I_{xG} = (LR^3/12) = 26.66\ m^4$.

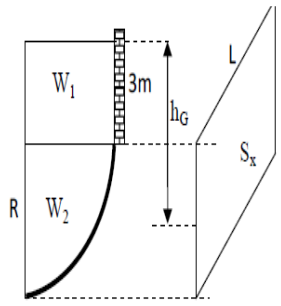
$h_p = 5.267\ m$

Vertical force: $F = \rho.g.W = \rho.g(W_1+W_2)$

$W_1 = 3R.L$ *et* $W_2 = \pi. R^2.L/4 = 1204.67\ KN.$

Application point: de FV is the center of gravity of the volume W :

$x_G = \frac{x_{G1}.S1 + x_{G2}.S2}{S1+S2}$



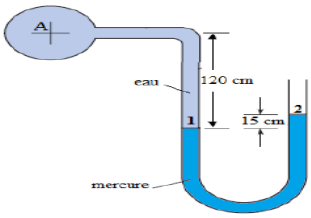
$$x_G = \frac{x_{G1}.S1 + x_{G2}.S2}{S1+S2}$$

$$= \frac{\frac{R}{2}.R.L + \frac{4R}{3\pi}\frac{\pi R^2}{4}}{R.L + \frac{\pi R^2}{4}}$$

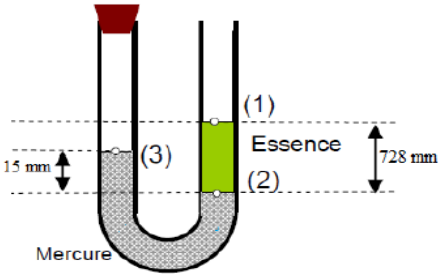
$$= 1.85m.$$

Exercises

Exercise 1: *Calculate the manometric pressure at point A in kPa, caused by the difference in mercury level, with a density of 13.6, in the U-shaped manometer shown in the figure opposite.*



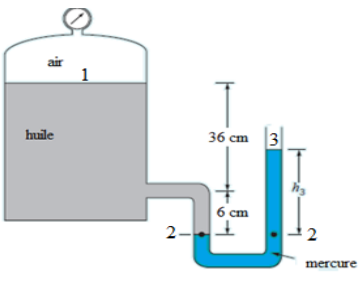
Exercise 2: *consider a U-shaped tube, closed at one end, containing two immiscible liquids. Calculate the pressure P3P_3P3 of the gas trapped in the closed branch. The following information is provided:" :* ρ_{Hg} *=13600 Kg/m³ et* $\rho_{essence}$ *= 700 Kg/m³, Patm =10⁵ Pa.*



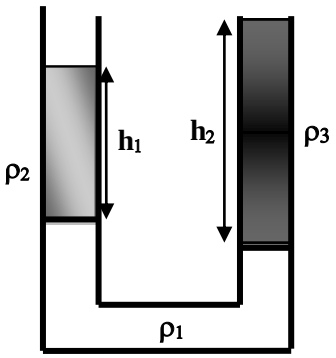
Exercise 3:

A closed, pressurized tank contains oil, and the pressure gauge reads 60 kPa. The density of the oil is d_h*=0.86 and the density of mercury is* d_{Hg} *=13,6*

- *Calculate the height difference of the mercury,* h_3



Exercise 4: *A U-shaped tube contains liquid $L_1$with a density of mass ρ_1. One side of the tube holds liquid L_2, with a volume $V_2=100\,cm^3$ and density of mass ρ_2, while the other branch contains liquid L_3 with a volume $V_3=200\,cm^3$and density ρ_3.*



1. *Calculate the distance between the two separating surfaces.*
2. *In which branch of the tube should a liquid L4with a specific gravity of 4 be added to ensure that the two separating surfaces are in the same horizontal plane?*
3. *Calculate the volume V_4 of the liquid L_4.*

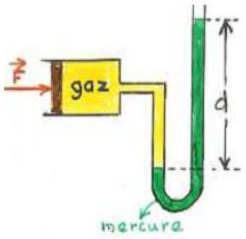
The following information is provided: : ρ_1=13600 Kg/m^3, ρ_2 = 1000 kg/m^3, ρ_3= 880 kg/m^3, ρ_4 = 700kg/m^3, g =10m/s^2,

The tube's diameter = 2 cm.

Exercise 5: *A gas is compressed in a cylinder, and its pressure is measured using a mercury manometer.*

Calculate:

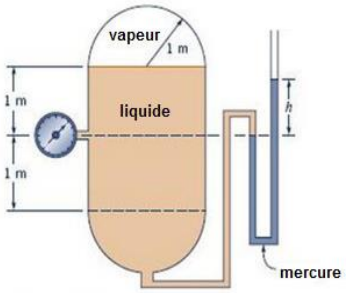
1. *The gas pressure, given that the mercury column height d=47 cm.*
2. *What is the relative pressure?*
3. *The force intensity F that the experimenter must exert on the piston, which has a cross-sectional area S=20 cm^2 and closes the cylinder."*



The following data is available : g=9,81m.s^{-2}, ρ_{Hg} = 13,6 x10^3 kg/m^3

Exercise 6: *A cylindrical tank contains a volatile liquid and its vapor. Given that the density of the liquid is 0.8 and the absolute pressure of the vapor is 120 kPa.*

1. *Calculate the relative pressure shown by the manometer.*
2. *Calculate the mercury height difference, H.*



Exercise 7: *Consider a U-shaped tube containing mercury with a density ρ_1. In one branch, water with a density ρ_2 is poured, and in the other branch, gasoline with a density ρ_3 is poured.*

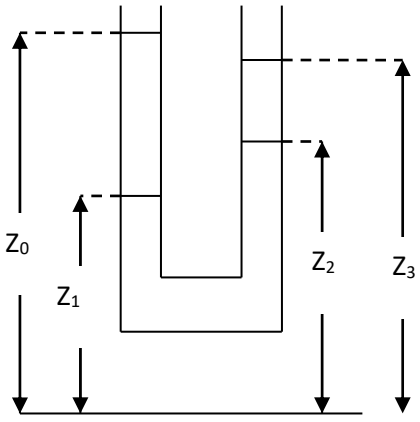
1. *Write the relationship that connects the different heights. z_0, z_1, z_2, z_3.*

1- *Calculate z_0, z_1, z_2, z_3 (figure opposite)*

The following data is available : z_0-z_1= 0.2 m,

z_1+z_2 = 1 m, z_3-z_2 = 0.1 m,

ρ_1 = 13600 kg/m^3, ρ_2 = 1000kg /m^3, ρ_3 = 700kg/m^3



Exercise 8: *A U-shaped tube with a cross-sectional area of 1 cm², open at both ends and containing water, has a volume of oil (V_{oil} = 6 cm^3) added to one of its branches.*

1- Determine the height difference between the water surface and the water/oil interface, and then calculate the height difference between the two free surfaces. Starting from this equilibrium state, a volume of acetone (V_{ac}=10 cm^3) is added to the other branch. Determine

the height difference between the two interfaces: water-oil and water-acetone, and then calculate the height difference between the two free surfaces.

The following data is available: $\rho_{eau} = 10^3$ kg/m^3, $\rho_{huile} = 900$ kg/m^3, $\rho_{acétone} = 790$ kg/m^3.

Exercise 9: *As a first approximation, we consider the blood (ρ_b=1.2 kg/L) to be in static equilibrium. The pressure at the level of the heart is taken to be 100 millimeters of mercury.*

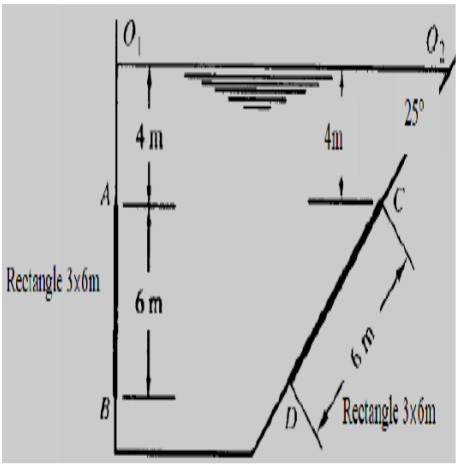
Calculate the hydrostatic pressure of the blood in the following two cases:

1. *At the level of the feet, 120 cm below the heart.*
2. *At the level of a cerebral artery, located 60 cm above the heart.*

Exercise 10:

A tank is equipped with two rectangular valves (3 m × 6 m) located on faces AB and CD.

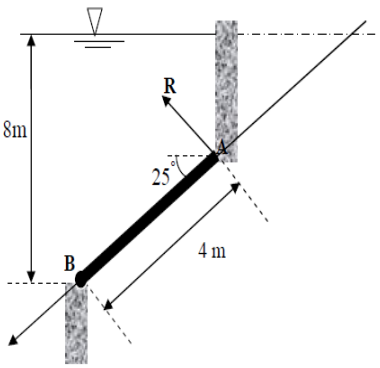
- *Calculate the pressure force on the valves AB and CD.*
- *Determine the center of pressure on the valves AB and CD.*
- *The CD valve is replaced by a circular valve. What would be the diameter of the circular valve required to maintain the same force calculated in part 1?*
- *Determine the new center of pressure on the CD valve.*



Exercise 11

The diagram below shows a door AB hinged at point B, with a width of b = 3 m and a length of L = 4 m. The door allows for the emptying of a large tank.

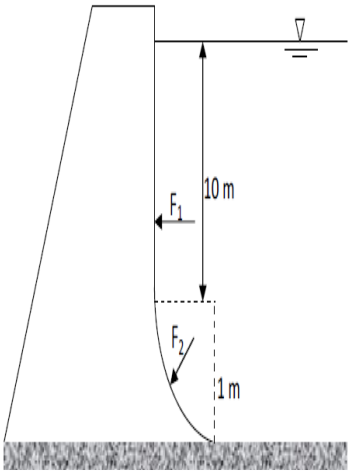
1. *Calculate the hydrostatic force acting on the door and determine its point of application.*
2. *Calculate the force R required to open the door.*
3. *If the weight of the door AB is 200 kN, what is the force **R** necessary to open the door?*



4. *If the weight of the door AB is 200 kN, what is the force **R** required to open the door?*

Exercise 12 *: A dam consists of a vertical surface 10 meters high, topped by an arc with a radius of 1 meter.*

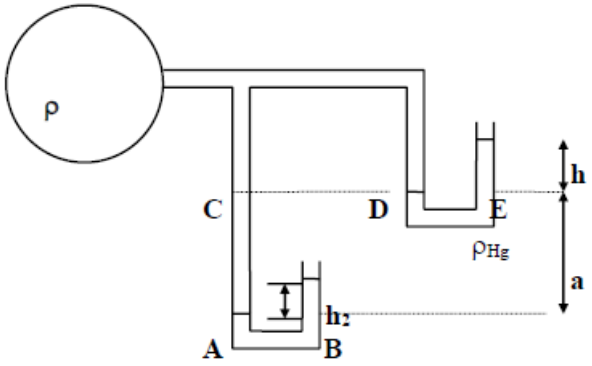
1. *Calculate the horizontal component (Rh) of the hydrostatic force exerted on the dam per unit of length.*
2. *Calculate the vertical component (Rv) per unit of length.*
3. *Calculate the norm of the resultant force and the angle it makes relative to the horizontal.*



Additional exercises

Exercise 1:

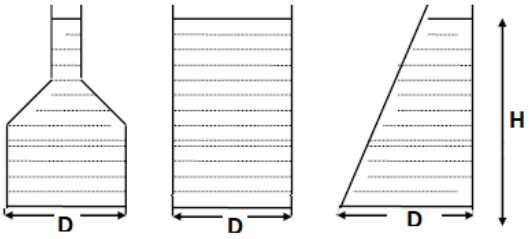
Determine the difference in height of the mercury in the system shown in the figure below.



Answer: $h_2 - h_1 = \rho.a / \rho_{Hg}$

Exercise 2:

Calculate the pressure forces exerted by the water on the bottoms of the containers shown in the figure below, given that the diameters of the bottoms and the water depths are D = 50 cm and H = 120 cm, respectively.



Answer : $F_V = \rho g V$, $V = H\pi D^2/4$, $F_V = 2356\ N$

Exercise 3 :

1- ***Find the height of the free surface*** *of water when 0.02 m³ of water is filled in a conical tank with the following dimensions:*

- *Height of the cone, h=0.5 m*
- *Radius of the base, r=0.25 m*

2- ***How much additional water*** *is needed to fill the tank completely?*

3- ***If the tank contains 30.5 kg of oil****, calculate the density of the oil.*

Answer :

1 $\boldsymbol{.V_{Con} = \frac{\pi r^2 h}{3} = 0.0327\ m^3}$

2. ***Volume of water that can be added****:*
The volume of water already in the tank is 0.02 m³, so the volume needed to fill the tank is:

$V_{add} = V_{con} - 0.02 = 0.0327 - 0.02 = 0.0127\ m^3$

Height of the free surface*:*
From the figure, we know that the ratio of the radius to height remains constant as the tank is filled. Using the relationship $r_0/0.25 = h_0/0.5$.

we can find the height h0 corresponding to the volume of water in the tank. Solving for h_0 : $h_0 = 0.364\ m$

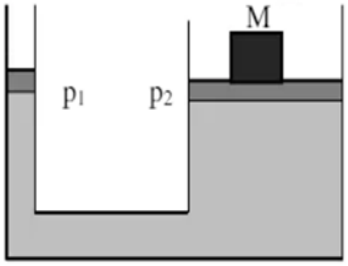
The free surface height is the difference between the total height of the tank and the height filled with water: Free surface height $= 0.5 - 0.364 = 0.136\ m$

3. ***Density of the oil****:*

Given the mass of oil moil=30.5 kg, the density of the oil can be calculated using the formula: $\boldsymbol{\rho_{oil} = \frac{m_{oil}}{V_{con}} = 932,7\ Kg/m^3}$

Exercise 4 :

A hydraulic press, as shown below, consists of two pistons, $P_1$and P_2located in the same horizontal plane. The liquid is incompressible, with areas S_1=10 cm^2 and S_2=1000 cm^2.



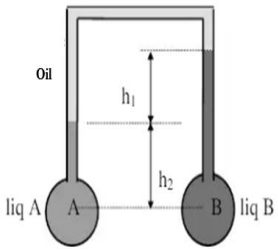
1. *What force must be exerted on P_1 to lift a mass M=1000 Kg placed on P_2?*

2. *The same question if piston P_1 is located 1 meter below piston P_2 (ρliq=1000 kg/m^3).*

Answer : *1-* $F_1 = \frac{S_1}{S_2}.M.g$, *2-* $F_1 = \left(\frac{M.g}{S_2} + \rho_{liq}.g.h\right).S_1$

Exercise : 5

The manometer, on the other hand, is used to measure the pressure difference between points A and B.
Given the following data:
h_1=4 cm, h_2=8 cm
ρoil=980 kg/m^3, ρ_{liqA}=1000 kg/m^3, ρ_{liqB}=1200 kg/m^3



1. *Determine the pressure difference between A and B.*

Answer : $P_A - P_B = [(\rho_A.h_2 + \rho_{oil}.h_2) - \rho_B(h_1 + h_2)].g$

Chapiter 2 : Archimedes Push

Chapiter 2 : Archimedes Push

I. Theorem: *Any body immersed in a liquid or in a gas in equilibrium undergoes from it a force called* ***Archimedes Push****.*

II Propreties of Archimedes Push

The characteristics Archimedes Push are:

The sense : Ascending or directed from the bottom to the top.

The direction : vertical

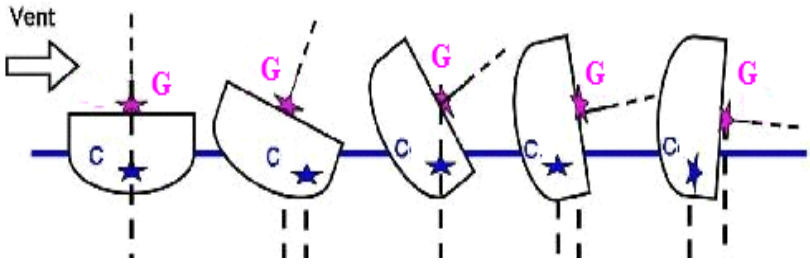
II.1 Application

Archimedes force *is applied to the center of gravity of the displaced liquid. It is called the thrust center or the center of the hull.*

The hull *is the submerged volume of the float (volume located below the flotation plane schematized below). The thrust of the liquid passes through the center of gravity of the volume of the hull. This is the thrust center or hull center.*

The center of gravity *is the point of concentration of the different forces that allow a body to find itself in balance. The center of gravity does not move.*

The center of the hull, in the case of a ship, for example*, or also the center of gravity of the fluid displaced by the hull, moves as a function of the submerged hull surface. The diagram, below, represents five boat positions seen from the rear with the different displacements of the center of hull C.*

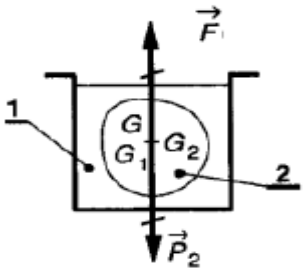


For a body of homogeneous volume weight and fully immersed, thrust is confused with the center of gravity of the body. The same is not true for floating bodies

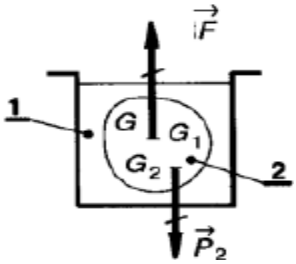
Notes:

- *If the fluid in which a solid is immersed is homogeneous, the center of thrust C of the fluid on the solid coincides with the center of gravity G_1 of the displaced fluid.*

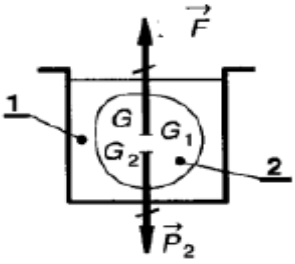
- *If the immersed solid is homogeneous, its center of gravity G_2 coincides with that of the displaced fluid G_1 and its equilibrium position is indifferent.*
- *If the immersed solid is not homogeneous (example: ship), the center of gravity G_2 of the solid is not confused with the center of gravity G_1 of the displaced fluid, the equilibrium of the solid is ensured, in this case, only in the position for which the center of thrust C is above, on the same vertical, as the center of gravity G_2 of the solid.*



1. ***Homogeneous fluid.***
2. ***Homogeneous solid***
3. ***Indifferent Equilibrium***



1. ***Homogeneous fluid.***
2. ***Non-homogeneous solid.***
3. ***Non-equilibrium position***



1. ***Homogeneous fluid.***
2. ***Non-homogeneous solid***
3. ***Non-equilibrium position.***

II.2 Intensity: *It is equal to the weight of the fluid. ρ_{liq} being the density of the liquid. The volume of the liquid being the volume of the displaced liquid or the volume of the immersed part of the body.*

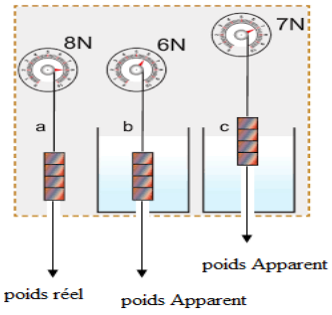
Note: *The Archimedes thrust depends on the nature of the liquid, in which the body is immersed, and the volume of the immersed body.*

II.3 Definitions

Immersed body: *A body is immersed in a liquid if its entire volume is located below the free surface of the liquid.*

Floating body: *A body is said to be floating if only part of this body is located below the free surface of the liquid.*

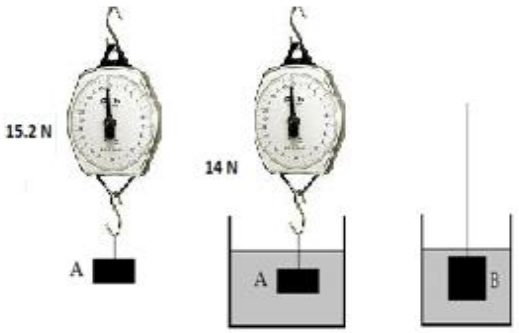
Real weight: *This is the weight of the body.*
Apparent Weight: *This is the weight of the submerged part of the object.*



Example

- What do the14 N et 15,2 N shown in the figure opposite represent ?

- We give ρ_{liq} = 1,2 g / cm^3. The volume of B is 200 cm^3. g = 10N / kg. What is the Archimedes' Push exerted by the liquid on bodies A and B ?

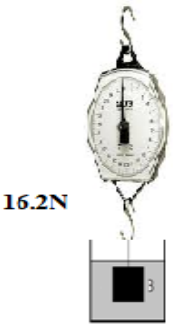


-

Compare the values found and then deduce the volume of A. ***B is completely immersed in another liquid with ρ_{liq} = 0,9 g / cm^3.*** *The dynamometer indicates 16.2 N.*

a- *Calculate the Push of this liquid on B.*

b- *Deduct the real weight of B.*



Solution

- 14N represents the apparent weight P_{App} et 15.2 N represents the actual weight of the object P.

The Archimedes thrust exerted on the body $F_{A/A}$ = real weight of A - Apparent weight of A = 15.2 – 14 =1.2 N

The Archimedes thrust exerted on Body B

- $F_{A/B}$ = 200.10^{-6}.1200.10 = 2.4 N.

- $F_{A/B} > F_{A/A}$

$F_{A/A} = \rho_L.g.Vi_A$, Vi_A is the submerged volume of the body

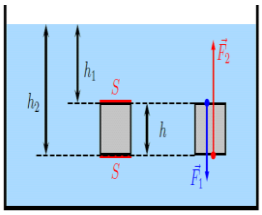
$F_{A/A} = \rho_L.g.V_A$ $V_A = 10^{-4}\ m^3$

$F'_{A/B} = \rho_{L.}g.V_B = 900.100.200.10^{-6} = 0.9N$

The real weight of body B PBreal = $Pbréel = F'_{A/B} + PApp = 0.9 + 16.2 = 17.1N$.

III.Theoretical establishment of the Archimedes formula :

Either a body in the form of a parallelepiped with a base S and a height h, immersed in a liquid with a density ρ_{liq}. The upper face is at a depth h_1, the lower face at a depth $h_2 = (h_1+h)$. The hydrostatic pressure at depth h_1 worth : $P_1 = \rho_{liq} \cdot g \cdot h_1$. In h_2, it is worth: $P_2 = = \rho_{liq} \cdot g \cdot h_2$



The liquid exerts the force F_2 on the lower face so that : $F_2 = P_2 \cdot S = \rho_{liq} \cdot g \cdot h_2 \cdot S$

The modulus of the force F_1, exerted on the upper face is: $F_1 = P_1 \cdot S = \rho_{liq} \cdot g \cdot h_1 \cdot S$.

The pressure P_2 being greater than P_1, $F_2 > F_1$. The resultant force is : $F_A = F_2 - F_1$, directed upwards, of norm : $F_A = \rho_{liq}.g.S.(h_2 - h_1) = \rho_{liq}.g.S.h = \rho_{liq}.g.V.$ V being the volume of the body.

F_A is the Archimedes force, It is the force caused by the hydrostatic pressure in the vertical direction.

III.1 Flotation Principle (Flotation and immersion conditions)

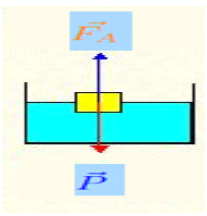
The flotation plane is the plane of the free surface of the liquid. This plane intersects the float along a surface called flotation, the contour of which is called the waterline.

Let be a homogeneous body with a density of mass ρ, completely immersed in a homogeneous liquid of density ρ_L. If we dive and then abandon the body, in a liquid, it is subjected to two forces : Its weight et and The push of Archimedes (F).

III.1.1 How to check if a body will sink or float?

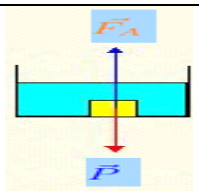
If the weight is smaller than the Archimedes thrust. The body will rise upwards, it is said that the body floats.

Apparent Weight < 0 : positive buoyancy



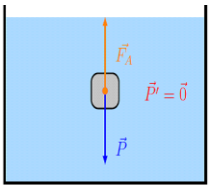
If the weight is greater than Archimedes' thrust. The body will descend to the bottom, it is said that the body sinks.

Apparent Weight > 0 : négative buoyancy



If the weight is equal to the Archimedes thrust. The body remains motionless in equilibrium.

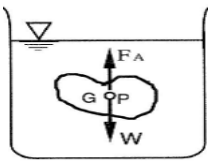
Apparent Weight = 0 : zero buoyancy



Note: *If the 2 specific masses are very different, the above law is applied. If they are close, it is necessary to calculate the weight of the solid and compare it with of the Archimede Pushs.*

III.1.2 Stability of the balance of a immersed body

A body is in equilibrium if the weight P and the Archimedes force are equal, opposite and located on the same vertical line. If the body remains static, it is in balance. Otherwise, a movement results.



Corps immergé

Stability is defined as follows :

If a body is inclined at an angle relative to the vertical, the body is subjected to a straightening torque which causes it to rotate until it returns to the initial position. The instability is therefore defined by a torque which tends to increase the inclination.

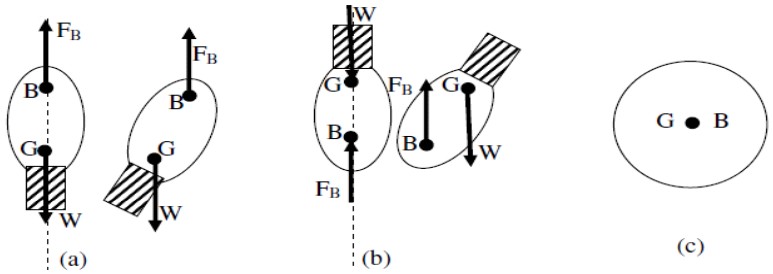
Case where P = G (case usually encountered if the body is homogeneous)

When there is the equilibrium, what kind is?

A submerged or floating body is said to be stable if it returns to its initial position after displacement.
The relative position of the center of gravity and the center of the hull determines the state of stability of a submerged body.

a. Stability of the eqilibrium of the submerged bodies

Either a completely or partially submerged body. The thrust of Archimedes is equal to the weight of the body; moreover, the center of gravity, G, and the center of thrust, C, are on the same vertical.



(a) représente un équilibre stable, (b) un équilibre instable, (c) un équilibre neutre.

For a state of equilibrium (F_A = P) the pushing force is equal to the weight of the balloon. If a slight angular displacement is made in the clockwise direction, the torque (F_A, P) will tend to return the body to its initial position, it is the stable balance of the figure (a), where the center of gravity is lower than the center of hull.

For the figure (b) where the center of gravity is higher than the center of hull, if an angular displacement is made in the clockwise directio, the couple (F_A, P) will also be in the same direction and the body does not return to its initial position, it is unstable equilibrium. When the center of gravity is superimposed with the center of hull (figure(c)), the balance is said to be neutral, because the couple (FA, P) will zero.

In summary

a- Stable Equilibrium : when (FA = P) et « G » is lower than « C »

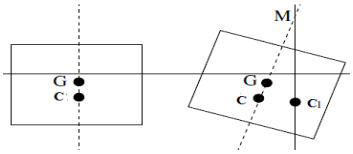
b- Unstable Equilibrium : when (FA = P) et « G » is higher than « C »

c- Neutral Equilibrium: quand (FA =P) et « G » et « C » superimposed.

b. Stability of floating bodies

As we have already mentioned, a floating body is in stable equilibrium if its center of gravity Comme nous l'avons déjà mentionné, un corps flottant est en équilibre stable si son centre de gravité (G) is located below its thrust center. However, some floating bodies can be

in equilibrium, even if this condition is not verified (G above C). The equilibrium of the floating bodies is a function of the relative position of the Metacenter «M», relative to the center of gravity « G » and from the center of carene «C».

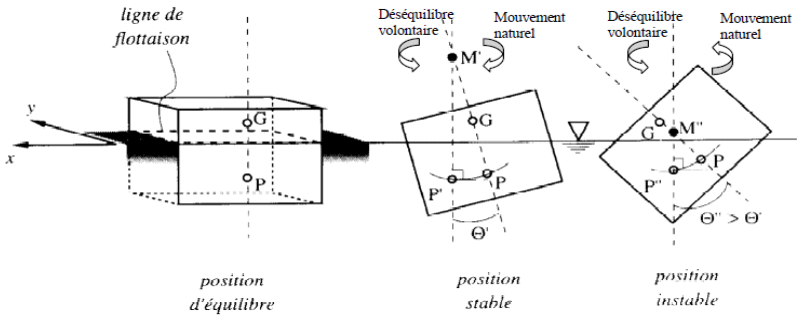


The Metacenter: *Let be a floating body with a center of gravity «G» and center of hull C. If the body makes a small angular displacement, the center of the hull moves to «C_1».*

The metacenter is defined as the center of oscillations of the floating body or the center of the arc «CC_1». The metacentric distance « MG » is the distance between the metacenter and the center of gravity of the body. A float in equilibrium undergoes small oscillations, or rotational movements, which can be broken down into two movements. This facilitates the study of certain movements, namely:

- *A rotation about a longitudinal axis of the float called «Roulis». The latter is carried out around what is called «Roulis metacenter», noted MR.*
- *A rotational movement about a transverse axis of the float called «Tangage». This is carried out around the «métacentre de Tangage», noted MT.*

In the following example, the stable equilibrium position is checked for an angle Θ'. The unstable position corresponds to the angle Θ'', with Θ'< Θ''. So there is a limiting angle Θ limit of switching between the stable state and the unstable.



The stability of a float is studied by comparing the metacentric CMR and CMT, which separate, respectively, the center of push «C» from the metacenter of Roulis «MR» and the metacenter of Tangage «MT» at the distance «CG» separating the push center «C» from the center of gravity G from the float (which represents the position of the center of gravity G of

the float with respect to the two metacenters MR andMT). The metacentric distances CMR et CMT are determined by the following:

$$CM_R = I S_{x'x} / V_{hull}$$

$IS_{x'x}$ is the moment of inertia of the flotation S with respect to the longitudinal axis x'x, passing through O, the center of gravity of S. V_{hull} is the volume of the hull.

$$CM_T = I S_{y'y} / V_{hull}$$

$I S_{y'y}$ the moment of inertia of the flotation S with respect to the transverse axis y'y, passing through O the center of gravity of S. Once the distance CG determined, three cases of figures can appear, by comparing the three distances, namely CG, CMR et CMT.

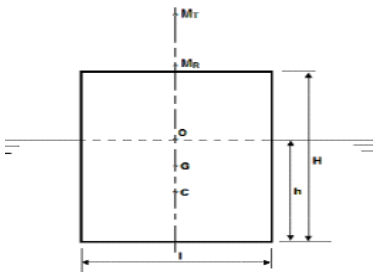
If CG < CMR < CMT, so the center of gravity G is lower than the two metacenters MR and MT, the float is in stable

If CG > CMT > CMR, so the center of gravity G is higher than the two metacenters MR et MT, the float is in unstable equilibrium.

if CMR < CG < CMT, so the center of gravity G is located between the two metacenters MR et MT, the float is in mixed equilibrium (sometimes stable, sometimes unstable).

Application example

Let be a rectangular parallelepiped shaped float of dimensions L.l.H, immersed from a height h as shown in the figure opposite.



1. Determine the position of the center of push C.

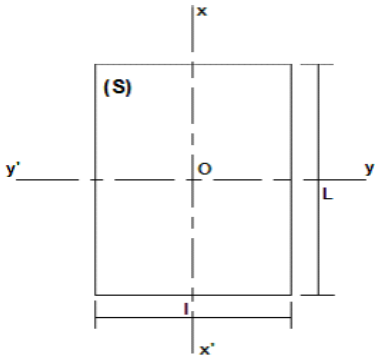
2. Calculate metacentric distances CMR et CMT.

3. Study the stability of this float.

Solution

1. *The push center is located at the center of gravity of the hull, rectangular parallelepipedal shape and dimensions L. l.h. donc OC = h / 2.*

Calculate the metacentric distances CMR and CMT.



*Let **S** be the waterline surface (surface of flotation), which is rectangular in shape with the following dimensions:*

- ***L**: the length of the flotation surface*
- ***l**: the width of the flotation surface*

*The axis **x'x** is the longitudinal axis of the flotation surface **S**, passing through its center of gravity **O**.*

*The **relative metacentric distance CMR** is the distance between the **center of buoyancy OC** and the **metacentre MR**, measured with respect to the **longitudinal axis of the flotation surface**.The **transverse metacentric distance CMT** is the distance between the **center of buoyancy OC** and the **metacentre MT**, measured along the **transverse axis of the flotation surface**.*

*To do this, you must use the **moment of inertia of the flotation surface S** with respect to the **longitudinal axis x'x**, that is, the **moment of inertia of a rectangle** relative to an axis **parallel to its length L** and passing through its **center of gravity O**.*

$I_{x'x}$ *: It is the **moment of inertia of the flotation surface S** with respect to the chosen axis x'x, It is given by the formula:* $I_{x'x} = \frac{l.h^3}{12}$

*where lll is the **width of the flotation surface**, and hhh is the **height of the hull (carène)**.*

*The **hull volume Vhull** is equal to L.l.h, where:*

is the length,l is the width of the flotation surface, andh is the height of the hull.

*The axis **y'y** is the **transverse axis** of the flotation surface **S**, passing through its **center of gravity O**.*

.

$CM_R = I_{x'x} / V_{carène}$

$CMT = I_{y'y} / V_{carène}$, $I_{y'y} = \frac{L.h^3}{12}$

2. *For the study of the stability of the float, the distance is determined CG, which we will compare to the metacentric distances CMR et CMT already calculated. CG = (H / 2) – (h / 2)*

Digital application: *we give : L = 8 m ; l = 4 m ; H = 6 m ; h = 4 m.*

We get: CMR = 42 / 12 . 4 = 0, 33 m ; CMT = 82 / 12. 4 = 1, 33 m et CG = 3 – 2 = 1 m.

CG = 1 m < CMR = 0,33 m < CMT = 1,33m

The center of gravity G is between the two metacenters MR et MT, the float is, therefore, in mixed equilibrium.

IV. Exercises

Exercise 1: *A solid of parallelepipedal shape with a density of mass 0.6 g/cm³ et de dimensions L = 20 cm, l = 10 cm et h = 4 cm, immersed in the ¾ when immersed in a container with alcohol. The solid is homogeneous. Calculate:*

1. *The volume of alcohol displaced, the mass of the solid and the density of the alcohol*

2. *To fully strengthen the solid, a force of intensity F is exerted on it. Calculate the new Archimedes thrust, which is exerted on the solid in this case, and calculate F.*

Exercise 2: *A homogeneous cubic solid, on side a, floats on the separation surface between two immiscible liquids, water and a hydrocarbon of density of mass d = 0,75, in relation to water. The solid is completely immersed and its lower base is at the distance a/5, below the separation surface of the two liquids.Faire un schéma et déterminer les forces exercées sur le solide.*

1. *Give the density of the solid as a function of d and the density of the water.*

Exercise 3:

a. *We consider a U-shaped tube of cross section s =1 cm², containing water and open to the open air, in the branch (2), we pour 5 cm³ of density of mass oil ρh_{ui} . Either h_1 the oil height, at equilibrium the difference in level between the two free surfaces is h. Represent the diagram corresponding to this tube and then determine the elevation difference h.*

b. *We introduce into the branch (2), un homogeneous solid having the shape of an edge cube a = 5mm and density of mass ρ_{solide} Show that the solid floats on the separation surface of the two liquids, then calculate the height z of the part of the cube that is bathed in oil. We give: $\rho huile = 0.85\ g/cm^2$, $\rho eau = 1\ g/cm^2$ et $\rho_{solid} = 1\ g/cm^2$.*

Exercise 4: *A wooden ball, of mass M = 195 g, is suspended from the lower end of a spring. This ball is immersed in water until the 1/3 of its total volume. At equilibrium, the spring, of negligible mass and stiffness $k = 50\ N.m^{-1}$, lengthens from $\Delta l = 1,9$ cm.*

1. *Calculate the value of the spring tension.*

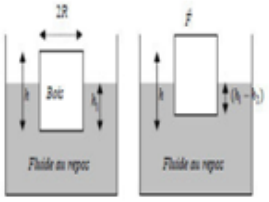
2. *Represent the forces exerted on the ball and write the equilibrium condition of the ball, deduce the value of the Archimedes push exerted on this ball*

3. *Determine the submerged volume of the ball, its total volume and the density of the wood.*

4. *The spring is abruptly cut off from its lower end. Indicate, justifying the answer, the state of flotation of the ball and then calculate the submerged volume of the ball.*

Exercise 5: *A metal cube of 15 cm from the side is suspended by a rope. The cube is immersed, half in the density oil 800 kg/m^3 and half in the water. If the density of the metal is 2640 kg/m^3. Represent the forces exerted on the cube and then find the tension force of the string.*

Exercise 6 : *A homogeneous wooden cylinder, with a density of ρ_b, of radius and height h, floats on a fluid at rest of density of mass ρ_f. When it is in equilibrium, its axis is vertical and it is immersed on a depth h_1 (see figure).*



1. *Determine the density of the cylinder ρ_b depending on h, h_1 and the density of mass of the fluid ρ_f.*

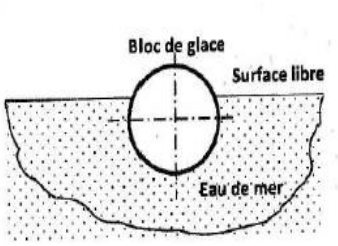
2. *A force F is exerted on the cylinder to lift the cylinder by a height h_2 (see figure 2).*

3. *Determine the modulus of the force F en fonction de R, h_2, g and the density of mass of the fluid ρ_f.*

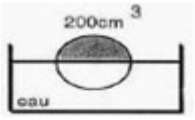
Exercise 7: *A homogeneous solid in the form of a parallelepiped with a density of mass 7.1 g/cm^3, of section and height h = 15 cm, is immersed in two immiscible liquids in equilibrium. Liquids are mercury (13600 Kg/m^3) and salt water (1100 Kg/m^3).*

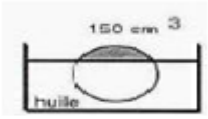
1. *Represent the forces applied on the solid .*
2. *Calculate the immersion height x0 of the solid in equilibrium.*

Exercise 8: *A spherical block of ice floats on the surface of the sea water. Sea water has a density of mass* ρ_w = 1025 Kg/m^3. *Ice has a density of mass* ρ_{ice} = 995 Kg/m^3. *Determine the fraction of the submerged volume* ($V_{submerged}/V_{sphere}$).



Exercice 9: *We consider a hollow steel sphere (*ρ *=7.85g/cm3), diving in the water plongée (*ρ_1 *= 1g/cm^3). The sphere floats letting emerge a volume of 200 cm^3. Plongée dans l'huile (*ρ_2 *= 0, 8 g/cm^3), it floats letting emerge a volume of 150 cm^3. Determine the weight of the sphere and its outer volume.*





Exercice 10: *A homogeneous parallelepipedal solid base surface* S_1 = 100 cm^2 *and height* h_1 = 10 cm, *has a density of mass* ρ_1 = 0,8 g.cm^{-3}. *This solid is immersed in a container containing water of density of mass* ρ.

1. *By comparing the density of mass* ρ *and* ρ_1 *shown that the solid will float.*
2. *Establish the relationship expressing the height e of the solid that emerges above the water* $e = h_1.(1-\rho_1/\rho)$

In a container with a base area S = 500 cm^2 the water reaches a height h = 40 cm. A volume of alcohol is added to the container, V_2 = 10,8 l *and density* d_2 = 0,8. *Alcohol is miscible in water.*

1. *Calculate the density of the mixture.*
2. *Calculate the new height of emergence of the solid after verifying that this solid does not flow.*
3. *What needs to be done To further immerse the solid ?*

We assume that the solid is connected to a wire attached to the bottom of the container. What tension is exerted on the wire if the solid is immersed at 9/10 then totally immersed.

Exercice 11: *We consider a hydrometer formed by a hollow cylindrical of length L = 400 mm and in diameter d, in which is placed a mass of lead at the level of its lower part. The center of gravity G from the hydrometer is located at a distance a =10 mm relative to the bottom. The density meter floats on the surface of a liquid with a density of mass ρ unknown. It is immersed up to a height h. When the hydrometer is placed in density of mass of water ρ = 1000 kg, the submerged height is h_0 = 200 mm.*

1. *What is the density ρ of the liquid if the submerged height h = 250 mm?*

2. *What is the density ρ_{min} that can be measured with this density meter?*

3. *Up to which value of the density ρ of the liquid the density meter remains in a stable vertical equilibrium ?*

4. *Give an example of a liquid in which there is a risk of having a stability problem.*

Exercice 12: *A solid consists of a steel cylinder with a cross-section S = 20 cm^2 and height h_1 = 2 cm and surmounted by a cork cylinder of the same section and height h_2 = 20 cm. The solid is placed in water$\rho_0 = 10^3$ Kg/m^3. The density of mass of the steel ρ_1 = 7800 kg / m^3, that of the cork is ρ_2 = 150 kg / m^3.*

1. *Show that the solid floats on the surface of the water.*

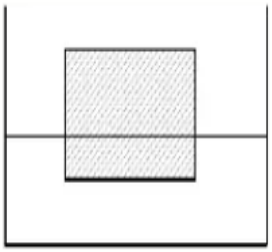
2. *In the vertical position (steel at the bottom), determine the submerged height and show whether this position is stable or unstable.*

V.Additional exercises

Exercise 1:*An open rectangular compartment, 10 m by 4 m in base and 5 m deep, has a mass of 54 tons and floats in fresh water.*

1. *How much is he sinking?*
2. *If the water has 5 m of depth, what weight of stones should be placed in the compartment to make it rest at the bottom?*

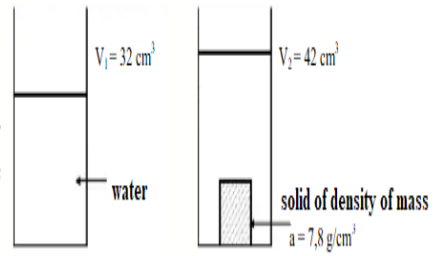
Exercice 2:*An object with a parallelepiped shape is immersed in a bucket of water. The object floats; it is 1/3 submerged (see figure). The volume of water displaced is 150 cm³. Calculate:*



1. *The mass of displaced water*
2. *Deduce the Archimedes' force.*
3. *The mass of the object (m_0).*
4. *Determine the volume of the object. Deduce its density.*

Exercise 3 : *During a practical lab session, a group of students from* ***Anador Middle School*** *conducted the experiment illustrated below on the left.*

1. *Calculate the volume of the object.*
2. *Calculate the mass of the object.*
3. *What is the weight of this object on Earth?*
4. *Does water exert a force on the object? What is this force called?*
5. *What is its magnitude?*
6. *What reading would a dynamometer attached to the submerged object display?*

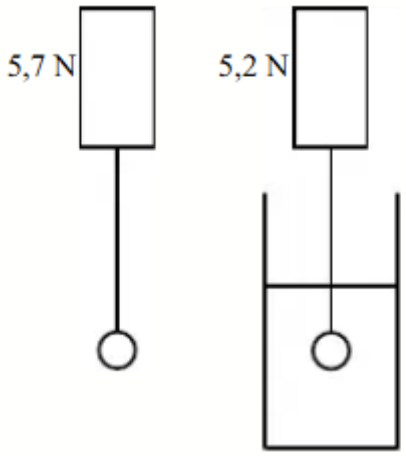


Exercise 4 : *A lead ball is suspended from a dynamometer.*

1. *Represent the weight of the ball*

   The ball is then immersed in water. A change in the reading of the dynamometer is observed.

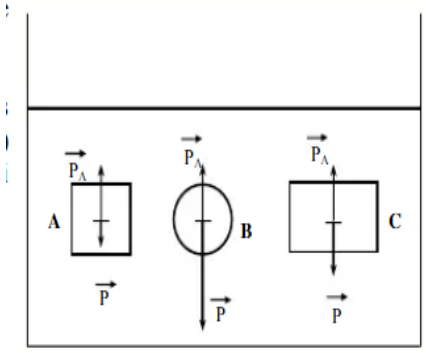
2. *What is the force responsible for this change in the dynamometer reading?*
3. *After defining all its characteristics, represent this force on the figure.*
4. *What does the dynamometer reading represent when the ball is immersed in water?*
5. *Calculate the volume of the lead ball.*
6. *Why does the dynamometer show a reading of 0 when the ball is immersed in mercury?*
7. *Calculate the volume of the ball that is submerged below the surface of the mercury.*
8. *We give* $\rho_{Hg} = 1.36\ g/cm^3$ *and* $\rho_{Pb} = 11{,}35\ g/cm^3$



Exercice 5

Three objects A, B, and C, all with the same volume, are fully submerged in a container filled with water. They are released at the same time. In the figure below, the main forces acting on each object at the moment they are released are shown. The scale is 1 cm = 1 N.

1. *For each object A, B, and C, say whether it sinks, rises to the surface, or stays in place (does not move). Justify your answer.*
2. *Determine the value of the buoyant force (Archimedes' force) acting on each object.*
3. *Calculate the volume V of object C.*



Chapiter 3 : Perfect fluid dynamics

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I. Definition of hydrodynamics : *It is the study of the physical properties of a fluid in motion under the influence of external forces, due to the force of gravity or to pressure differences.*

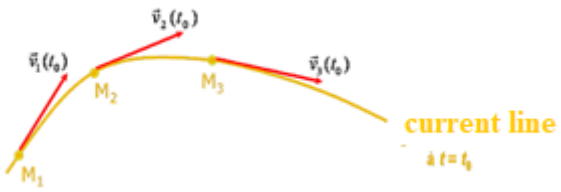
I.1 General definitions

The perfect fluid: *It is an ideal fluid in which the frictional forces are zero.*

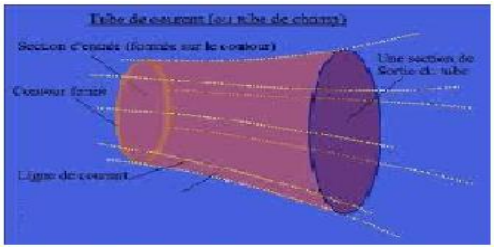
The real fluid: *is a fluid that manifests resistance to flow by friction.*

The permanent flow (Stationary)): *A flow is said to be stationary or permanent if the velocity field of the particles of the fluid as well as the pressure at a given point are independent of time.* ***Permanent movement*** $\Rightarrow \partial p / \partial t = 0$

Current line and tube: *The current line is called the tangent curve, at each of its points, to the velocity vector. A current tube represents an elementary part of the Fluid in motion, limited by current lines.*



Current line



Current tube

II. General equations of the dynamics of perfect fluids

II.1 Euler's equation: *Let be a fluid particle of volume* $dV = dx.dy.dz$, *located at a point with coordinates x, y and z, in an Earth's gravitational field g.*

According to the oz axis: *The forces that act on this volume element (dS.dz) are :*

1. *The force of volume:* $\rho.g\ (dS.dz)$ ***2.*** *The pressure forces* $P.dS$ *and* $(P+(\partial P/\partial z)dz).dS$

3. *The force of inertia (acceleration)* $\rho.\ d_{Vz} / dt\ .\ dS\ .\ dz$

Vz is the component of the velocity vector according to the direction z. the set of forces satisfies Newton's equation : $\sum \vec{F}_{ext} = m.a$. *The condition of balance of forces according to the direction oz is written :* $P.dS - \left(P + \frac{\partial P}{\partial Z}.dZ\right).dS - \rho.gdS.dz = \rho \frac{dv}{dt}\, dS.dz \quad \Rightarrow$

$$-\frac{\partial P}{\partial z} - \rho g = \rho \frac{dv}{dt}$$

Or per unit volume, we can write identically the condition of equilibrium of forces in the other directions, then in its vector form.

According to the axis ox:

$$P(x,y,z).dS - \left(P + \frac{\partial P}{\partial x}.dx\right).dy.dz = \rho \frac{dv_x}{dt}\, dx.dy.dz$$

$$P(x,y,z).dy.dz - \left(P + \frac{\partial P}{\partial x}.dx\right).dy.dz = \rho \frac{dv_x}{dt}\, dx.dy.dz$$

$$P(x,y,z) - \left(P + \frac{\partial P}{\partial x}.dx\right) = \rho \frac{dv_x}{dt}\, dx \quad \Rightarrow \quad -\frac{\partial P}{\partial x} = \rho \frac{dv_x}{dt}$$

According to the axis oy :

$$P(x,y,z).dS - \left(P + \frac{\partial P}{\partial y}.dy\right).dx.dz = \rho \frac{dv_y}{dt}\, dx.dy.dz$$

$$P(x,y,z).dx.dz - \left(P + \frac{\partial P}{\partial y}.dy\right).dy.dz = \rho \frac{dv_y}{dt}\, dx.dy.dz$$

$$P(x,y,z) - \left(P + \frac{\partial P}{\partial y}.dy\right) = \rho \frac{dv_y}{dt}\, dy \quad \Rightarrow \quad -\frac{\partial P}{\partial y} = \rho \frac{dv_y}{dt}$$

The total differential of p is:

$dp = \frac{\partial p}{\partial x}.dx + \frac{\partial p}{\partial y}.dy + \frac{\partial p}{\partial z}dz$ *Let's Replace* $\frac{\partial p}{\partial x}$; $\frac{\partial p}{\partial y}$ *and* $\frac{\partial p}{\partial z}$ *by their expressions*

$$dp = -\rho \frac{dv_x}{dt}.dx + \rho \frac{dv_y}{dt}.dy - \rho g.dz - \rho \frac{dv_z}{dt}dz$$

$$dp = -\rho dv_x \frac{dx}{dt} + \rho.dv_y \frac{dy}{dt} - \rho g.dz - \rho.dv_z \frac{dz}{dt}$$

$$dp = -\rho\, v_x dv_x + \rho.v_y dv_y - \rho g.dz - \rho.v_z dv_z$$

$\vec{v}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$, $\overrightarrow{dv}\begin{pmatrix} dv_x \\ dv_y \\ dv_z \end{pmatrix}$ $\vec{v}.d\vec{v} = v_x.dv_x + v_y.dv_y + v_z.dv_z$ So : $dp = -\rho vdv - \rho.g.dz$

$$dp + \rho vdv + \rho.g.dz = 0$$

1. *Pressure force per unit volume.*

2. *Inertial force per unit volume.*

3. *Volume Force per unit volume.*

The equations are called general equations of perfect fluid dynamics or Euler equations. Let's integrate Euler's equation, between 2 points of the liquid vein.

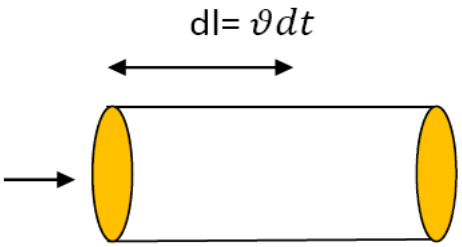
$\int_1^2 dp + \rho v dv + \rho . g dz = 0$ *Either :* $[P]_{p1}^{p2} + \frac{\rho}{2}[v^2]_{v1}^{v2} + \rho . g[z]_{z1}^{z2} = 0$

$$P_1 + \frac{\rho}{2}{v_1}^2 + \rho g z_1 = P_2 + \frac{\rho}{2}{v_2}^2 + \rho g z_2$$

II.2 Concept of Flow

2.1 Mass flow rate : *We call mass flow rate, the quantity,* $Q_m = \frac{dm}{dt} = \rho . S . V$ *which represents the mass of fluid passing through the section S of the pipe per unit time. the Unit is Kg/s. The flow rates are usually counted positively in the direction of :*

2.2 Volume Flow Rate : This is the volume *Dv = Sdl = Svdt, of fluid passing through a straight section of the pipe during the unit of time..* $Q_V = \frac{dV}{dt} = S . v$ *. For a perfect fluid, the speed v is the same at any point of the section.*



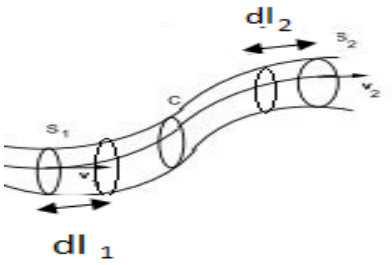
2.3 Principle of conservation of mass

The principle of conservation of mass is translated by the equality of the mass of incoming fluid by S1 between the moments t et t+dt with the mass of fluid leaving by S2, during this same period. $dm_1 = dm_2$ *. In the case of an incompressible fluid, the density of mass is constant. We have :* $dm_1 = dm_2 = \rho . S_1 . dl_1 = \rho . S_2 . dl_2 \quad \rho . S_1 . v_1 dt = \rho . S_2 . v_2 \, dt$

Where : $\rho . S_1 . v_1 = \rho . S_2 . v_2$.

The principle of conservation of mass is written :

$$\rho . S_1 . v_1 = \rho . S_2 . v_2$$



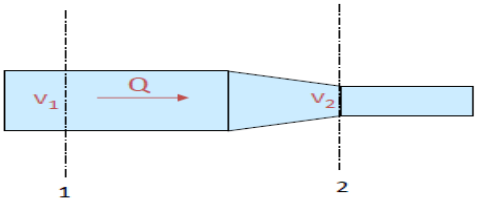
2.4. Continuity equation

In a pipe traversed by an incompressible fluid, the flow rate is the same regardless of the section considered. $Q_1 = Q_2 \Rightarrow S_1 . v_1 = S_2 . v_2$

Important consequence : *In a pipe traversed by an incompressible fluid, the speed of the fluid increases when the section decreases*

Example 1

Or a pipe whose area of section 1 is 0,100 m² and the speed of the liquid there is 1,20 m/s; the area of the section 2 is from 0,050 m². Calculate the speed of the current in 2 and the flow rate.



Solution

Let's apply the continuity equation between the sections 1 et 2 :

$$v_{.1} S_1 = v_2 S_2 \Rightarrow v_2 = \frac{v_1 . S_1}{S_2} = 2.4\ m/s$$

The flow rate is given by:

Qv = V.S = 2.4 x 0,05 = 0.120 m³/s

Qv = 2.4 x 0,05 = 0.120 m³/s

Example 2

Case of a current bifurcation where the section 1 has a diameter of 400 mm, the section 2 has a diameter of 200 mm and the diameter of the section 3 is 160 mm. If the speed in section 1 is 1,60 m/s and the one in the section 2 is of 3,9 m/s, calculate the flow rates Q1 , Q2 et Q3 as well as the speed at the section 3.

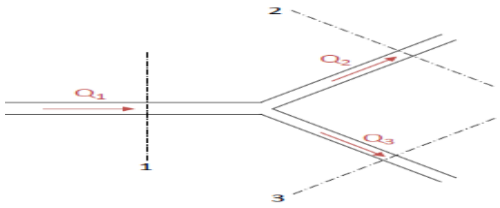
Solution

The flow rates are given by the continuity equation:

$$Q_{v1} = V_1 . S_1 = 1.6 \frac{\pi . 0.4^2}{4} = 0.201 m^3/s$$

$$Q_{v2} = V_2 . S_2 = 3.9 \frac{\pi . 0.2^2}{4} = 0.122 m^3/s$$

$Q_3 = Q_1 - Q_2 = 0.201-0.122 = 0.079\ m^3/s$



$Q_3 = 0.201 - 0.122 = 0.079\ m^3/s$

The speed at the section 3 is :

$$V_3 = \frac{Q_3}{S_3} = \frac{4.0.079}{\pi.(0.16)^2} = 3.93m/s$$

III. Energetics of perfect fluids in flow

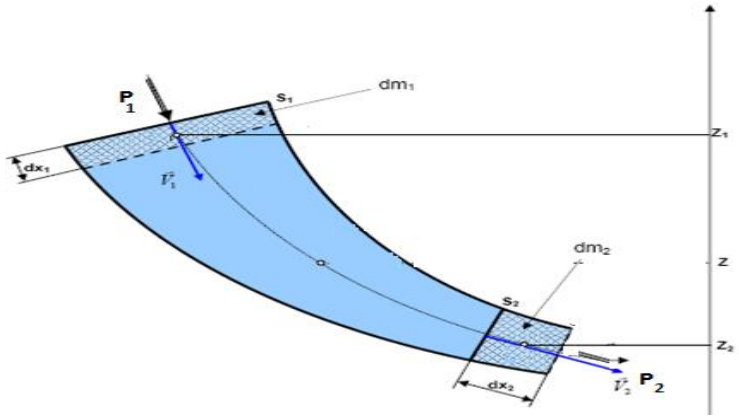
The Bernoulli equation can be obtained either by the conservation of the total mechanical energy for a perfect fluid or by the sum of the forces (Newton's principle).

III.1 Bernoulli's theorem

Bernoulli expresses the conservation of the energy of a stationary flow of an incompressible fluid, of density ρ, in a perfectly smooth pipe. In this flow, two points 1 and 2 belonging to the same trajectory are chosen.

Notations

- *P_1 and P_2 are the pressures of the fluid acting at the level of the sections S_1 et S_2.*
- *S_1 and S_2 are respectively the inlet section and the outlet section of the fluid.*
- *v_1 and v_2 are the flow velocity vectors respectively through the sections S_1 and S_2 of The pipe*
- *dx_1 and dx_2 are respectively the displacements of the fluid during the time interval dt.*
- *dm1 : incoming elementary mass during the time interval dt.*
- *dm2 : outgoing elementary mass during the time interval dt*
- *dV1 : incoming elementary volume during the time interval dt and*
- *dV2 : outgoing elementary volume during the time interval dt.*



a- Expression of different forms of mechanical energy

Form of energy	at the entrance	at the exit
Kinetic energy	$Ec_1 = \frac{1}{2} dm_1 {V_1}^2 = \frac{1.}{2} \rho . S_1 . dx_1 . {V_1}^2$	$Ec_2 = \frac{1}{2} dm_2 {V_2}^2 = \frac{1.}{2} \rho . S_2 . dx_2 . {V_2}^2$
Potential energy of gravity	$W_{Z1} = dm_1 . gZ_1 = \rho . S_1 . dx_1 . g . Z_1$	$W_{Z2} = dm_2 . gZ_2 = \rho . S_2 . dx_2 . g . Z_2$
Potential pressure energy	$W_{P1} = P_1 . S_1 . dx_1$	$W_{P2} = P_2 . S_2 . dx_2$

b- Expression of the principle of conservation of energy :

$$Ec_1 + W_{Z1} + W_{P1} = Ec_2 + W_{Z2} + W_{P2}$$

$$\frac{1.}{2} \rho . S_1 . dx_1 . {V_1}^2 + \rho . S_1 . dx_1 . g . Z_1 + P_1 . S_1 . dx_1 = \frac{1.}{2} \rho . S_2 . dx_2 . {V_2}^2 + \rho . S_2 . dx_2 . g . Z_2 + P_2 . S_2 . dx_2$$

$$\rho . S_1 . dx_1 . \left({V_1}^2 + g . Z_1 + \frac{P_1}{\rho} \right) = \rho . S_2 . dx_2 . \left({V_2}^2 + g . Z_2 + \frac{P_2}{\rho} \right)$$

According to the continuity equation: $\rho . S_1 . dx_1 = \rho . S_2 . dx_2$

$$\left(\frac{1.}{2} {V_1}^2 + g . Z_1 + \frac{P_1}{\rho} \right) = \left(\frac{1.}{2} {V_2}^2 + g . Z_2 + \frac{P_2}{\rho} \right) \rightarrow \left(\frac{1.}{2} \rho {V_1}^2 + \rho . g . Z_1 + P_1 \right) = \left(\frac{1}{2} \rho . {V_2}^2 + \rho . g . Z_2 + P_2 \right)$$

Note : *This last equation is valid only along the same trajectory of a particle.*

C. Energy interpretation of the terms of the Bernoulli equation

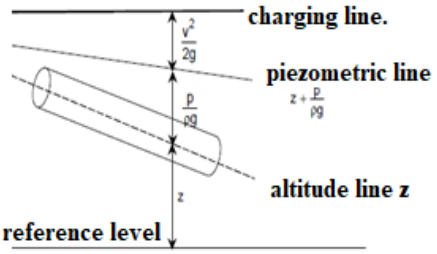
P_1, P_2 : *are the static pressures* $\frac{1}{2}\rho V^2$: *is the dynamic pressure*

$\rho.g.Z$: *is the hydrostatic pressure..*

We call "load height", the height $h = \frac{V^2}{2g} + . Z + \frac{P}{\rho.g}$. *This charge height is therefore constant along a trajectory of a particle*

$Z + P / \rho g$ *is called the piezometric load (or piezometric height).*

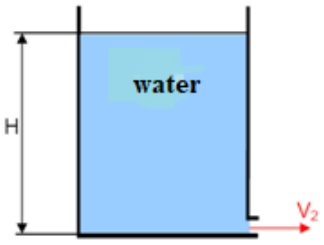
Bernoulli's theorem can then be interpreted graphically from the evolutions of the different heights along the circuit. As illustrated in the figure, below.



III.2 Applications of the Bernoulli equation

a. Liquid at rest : *In this case the speeds are zero; we will have* $P_1 - P_2 = \rho . G . (Z_2 - Z_1)$. *So, we find the fundamental equation of hydrostatics.*

b. Emptying a tank (Torricelli Formula) : *Consider a tank of large section* S_1 *empties through a small hole of section* S_2 *located on the side at the very bottom of the tank. We are looking for the speed of the fluid at the exit from the orifice.*



The application of the Bernoulli equation to the points 1 and 2, gives : $\frac{1}{2}\rho V_1^{\ 2} + \rho . g . Z_1 + P_1 = \frac{1}{2}\rho . V_2^{\ 2} + \rho . g . Z_2 + P_2$

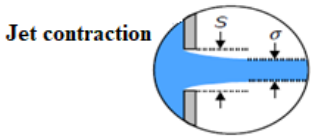
$P_1 = P_2 = P_{atm}$

$$\frac{1}{2}\rho V_1^{\ 2} + \rho . g . Z_1 + P_{atm} = \frac{1}{2}\rho . V_2^{\ 2} + \rho . g . Z_2 + P_{atm}$$

$$\frac{1}{2}\rho V_1^{\ 2} + \rho . g . Z_1 = \frac{1}{2}\rho . V_2^{\ 2} + \rho . g . Z_2$$

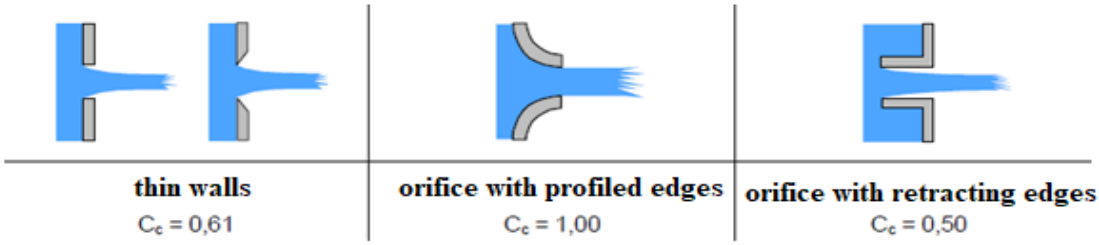
According to the continuity equation we have : $Q = S_1 . V_1 = S_2 . V_2 , S_1 \gg S_2 \Rightarrow \frac{V_1}{V_2} = \frac{S_2}{S_1} \ll 1$

so $V_1 \ll V_2$ *so* $V_2 = \sqrt{2g(Z_1 - Z_2)}$, $(Z_1 - Z_2) = h \Rightarrow V_2 = \sqrt{2gh}$



The volume flow rate is: $Qv = \sigma . V_2 = \sigma . \sqrt{2gh}$ $\sigma = C_c S$

The coefficient of contraction, Cc, depends on the geometry of the orifice. In general, Cc is determined experimentally and tabulated :

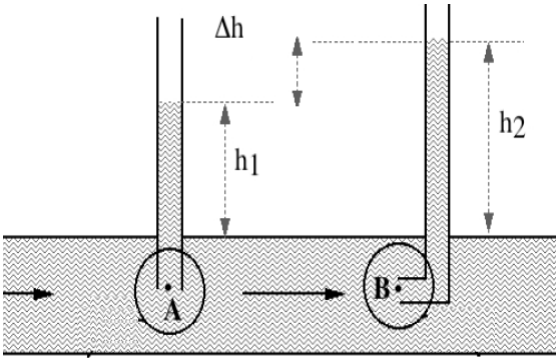


c. Flow velocity measurement: Pitot tube

By applying the Bernoulli equation between two points A and B :

$\frac{1.}{2}\rho {V_A}^2 + \rho . g . Z_A + P_A = \frac{1}{2}\rho . {V_B}^2 + \rho . g . Z_B + P_B$ *As* $Z_A = Z_B$ *et* $V_B = 0$,

so $P_B - P_A = \frac{1.}{2}\rho {V_A}^2$ ***(1)***

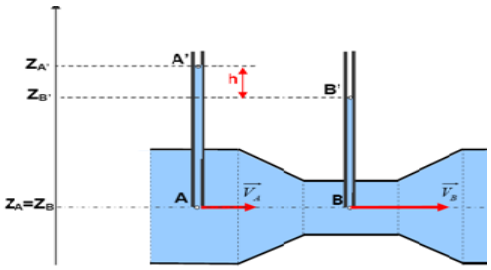


In the two vertical tubes, the liquid is at rest, so we apply the fundamental relationship of hydrostatics to it, we will have:

$$\left.\begin{array}{l} P_A - P_{atm} = \rho . g h_1 \\ P_B - P_{atm} = \rho . g h_2 \end{array}\right\} \Rightarrow P_B - P_A = \rho . g (h_2 - h_1) = \rho . g . \Delta h \qquad \textbf{(2)}$$

From the equations *(1) et (2) , we will have:* $\frac{1.}{2}\rho {V_A}^2 = \rho . g . \Delta h \Rightarrow V_A = \sqrt{2 . g . \Delta h}$

d . Venturi phenomenon



1. *Continuity equation:* $Q = S_A.V_A = S_B.V_B \Rightarrow V_B = V_A.\frac{S_A}{S_B} = \alpha.V_A$

2. *By applying the Bernoulli equation in A and B :*

$$\frac{1.}{2}\rho V_A^{\ 2} + \rho.g.Z_A + P_A = \frac{1}{2}\rho.V_B^{\ 2} + \rho.g.Z_B + P_B$$

we will have $Z_A = Z_B$ *donc :* $\frac{1.}{2}\rho V_A^{\ 2} + P_A = \frac{1}{2}\rho.V_B^{\ 2} + P_B \Rightarrow$

$$P_A - P_B = \frac{1.}{2}\rho.V_B^{\ 2} - \frac{1.}{2}\rho.V_A^{\ 2} = \frac{1}{2}.\rho\left(V_A.\frac{S_A}{S_B}\right)^2 - \frac{1.}{2}\rho.V_A^{\ 2} = \frac{1.}{2}\rho.V_A^{\ 2}.(\alpha^2 - 1)$$

$$\boxed{P_A - P_B = \frac{1.}{2}\rho.V_A^{\ 2}.(\alpha^2 - 1) \qquad (1)}$$

The fundamental relationship of hydrostatics between the points A and A' $\boxed{P_A - P_{A'} = \rho.g(Z_{A'} - Z_A)}$

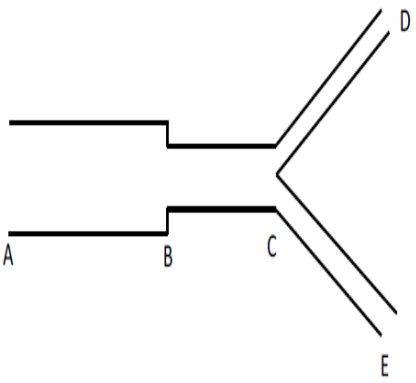
the fundamental relationship of hydrostatics between the points B and B' $\boxed{P_A - P_{B'} = \rho.g(Z_{B'} - Z_B)}$

$P_{B'} = P_{A'} = P_{atm}$ *et* $Z_A = Z_B$ *so* $\boxed{P_A - P_B = \rho.g.h \qquad (2)}$

According to (1) and (2) we get: $V_A = \sqrt{\frac{2.g.h}{(\alpha^2-1)}}$

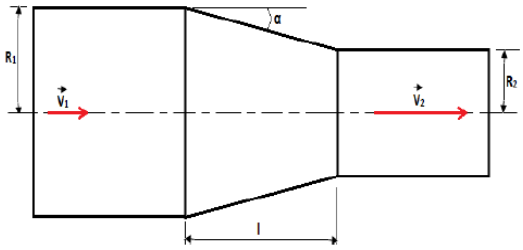
IV. Exercises

Exercise 1 : *Water flows at a uniform speed of 2 m/s in a pipe AB of d1 = 1,5 m diameter connected to a pipe BC of d2 = 1,2 m in diameter. At point C , the pipe separates into two parts. The first CD has a diameter of d_3 =0,8 m and carries one third of the total flow. The speed in the second CE is2,5 m/s. Calculate*



1. *The flow rate in AB .* ***2.*** *The speed in BC .* ***3.*** *The speed in CD* ***4.*** *The diamèter CE.*

Exercise 2 : *It is desired to accelerate the circulation of a perfect fluid in a pipe in such a way that its speed is multiplied by 4 .For this, the pipe comprises a convergent characterized by the angle α.*



1. *Calculate the ratio of the radius (R1/R2).*
2. *Calculate (**R1** - **R2**) as a function of l and α. Deduce the length l.*
   Digital Application: R1 = 50 mm, α = 15°.

Exercise 3 *An incompressible perfect fluid flows from a circular orifice located on the side of a tank with a volume flow rate qv=0,4 L/s. The diameter of the orifice is d=10 mm.*

1) Determine the flow velocity at the orifice.

2) State Bernoulli's theorem.

3) At what distance from the free surface is the orifice?

Exercise 4 : *Either a supposedly incompressible fluid (water) which flows through a venturi tube shown in the figure.*

Data : ρHg = 13.6 103 kg/m3 ρeau = 103 kg/m3 V1= 6m/s , D_1= 25 cm . We ask to calculate the diameter D_2 if p1= p2.

If this pressure (p2) is of 12N/cm², what is the value of Δh ?

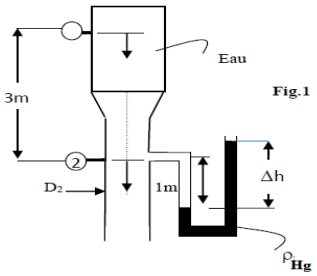
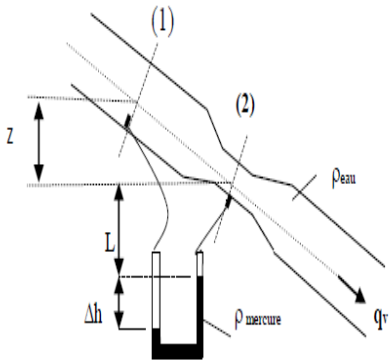


Fig.1

Exercise 5 : *A venturi tube is arranged on an inclined water pipe. The connecting tubes to the pressure gauge are filled with water .*

1-Calculate the vertical difference h of the pressure gauge as a function of V_1, S_1, S_2, ρeau, ρ_{Hg}

2-Calculate the volume flow rate Δh = 90 mm and D_1 200 mm et D_2 = 90 mm.

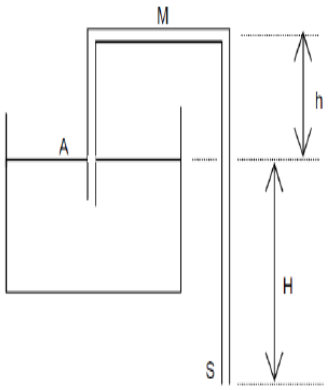


Exercise 6 : *Let be a siphon of diameter d fed by a container filled with water, of large dimension relative to d and open to the atmosphere.*

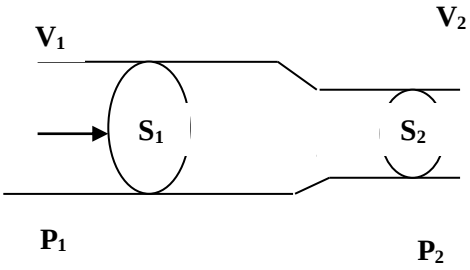
- ***a-*** *Calculate the average speed of the fluid in S and then the volume flow Qv of the siphon.*
- ***b-*** *Give the expression of the pressure PM at the point M as a function of h. Make a numerical application for h=0.4 m*

c- *h can it take any value?*

Digital Application:H = 3 m ; d=10mm ; Patm = 1 bar



Exercise 7 : *A horizontal tube has a circular cross-section of 20 cm in diameter, followed by a narrowing of 10 cm in diamètre. Knowing that the speed of the water in the main tube is 0,1 m/ s and the pressure is 10^3 Pa. Calculate the speed and the pressure of the water in the narrowing,, we give the density of mass of water* $\rho_{eau} = 1000Kg/m^3$

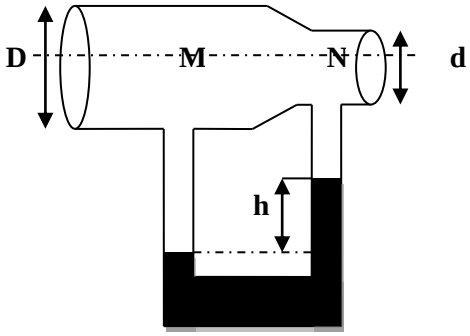


Exercise 8 : *The device on the other hand is used to measure the velocity VM of an incompressible fluid.*

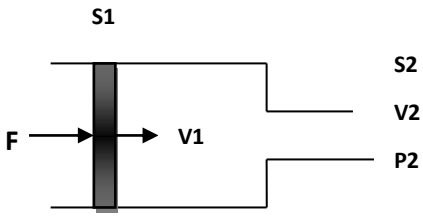
1. Express the speed V_N *as a function o*V_M*, D,d*

2. Express the pressure differenceP_N-P_M

3 Deduce therefrom the expression of the speed V_M *as function of* ρ_{eau},ρ_{Hg} *,h, D and d.*



Exercise 9 : *The figure opposite represents a piston which moves without friction in a cylinder of section* S_1 *and of diameter* d_1 *= 4cm filled with a perfect fluid of density* $\rho = 10^3\ Kg/m^3$.



The piston is pushed by a force F of intensity 62.84 N at a constant speed. The fluid can escape to the outside through a cylinder of section S_2 *and of diameter* d_2 *= 1cm at a speed V 2 and a pressure P 2 = Patm = 1 bar.*

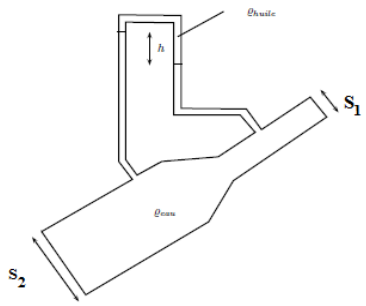
1*- Determine the pressure P1 as a function of F, Patm and d1.*

2*- By applying the Bernoulli equation, determine the flow velocity V2 as a function of* P_1*, Patm and* ρ*.*

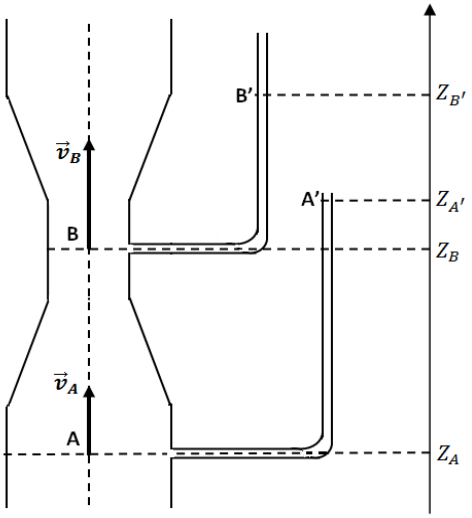
3*- Deducing the volume flow rate Qv .*

Exercise 10 : *Consider a flow with a flow rate Q, through a contraction. The pressures upstream and downstream of the contraction are measured using a manometer (see figure) containing density oil* $\varrho_h < \varrho_{eau}$.

The upstream and downstream sections are denoted respectively S_1 *and* S_2. *Determine the height h given by the manometer.*



Exercise 11 : *In the Venturi tube shown in the diagram below, the water flows from the bottom up. The diameter of the tube at A is* $d_A = 30$ *cm and at B it is* $d_B = 15$ *cm In order to measure the Pressure PA at point A and the pressure PB at point B, two water column manometers are connected to the Venturi. These piezometric tubes are graduated and make it possible to measure the Levels* $Z_A = 3{,}061$ *m and* $Z_{B'} = 2{,}541$*m respectively of the free surfaces A' and B'. It is assumed that the fluid is perfect and we give:*



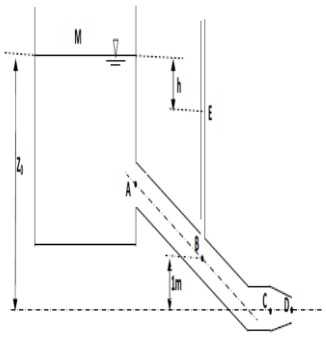
- *The altitude of section A:* $Z_A = 0$ *m,*
- *The altitude of the B section :* $Z_B = 50$ *m,*
- *The acceleration of gravity* $g = 9{,}81$ *m/s*^{2}
- *The pressure at the level of the free surfaces* $P_{A'} = P_B' = P_{atm} = 1$ *bar*
- *The density of lass of the water is* $\rho = 1000$ *Kg / m*^{3}

a. *Apply the fundamental relationship of hydrostatics between B and B', and calculate the Pressure* P_B *at point B. Similarly, calculate the pressure* P_A *at point A.*

b. *Write the continuity equation between points A and B. Deduce the flow velocity* v_B *as a function of* v_A.

c. *Write the Bernoulli equation between points A and B. Deduce the speed of flow* v_B.

Exercise 12 : *The figure opposite shows a large-sized tank filled with water, and whose level Z0 = 4m. AC is a pipe with a diameter D = 5 cm. At C is a short nozzle with an outlet diameter d = 2.0 cm. C and D are on the same horizontal. The water comes out of D in the open air. A tube is placed at B (piezometer) in connection with the pipe. The liquid is perfect.*



a- *Calculate the speed V_D of the water at the outlet of the nozzle. Calculate the flow rate Q*

b- *Deduce the speed V in the pipe AC. Calculate the pressure in B.*

c- Determine the difference in the h levels between the free surfaces of the tank and the tube.

V. Additional exercises

Exercise 1 : *Water flows through the vertical pipe shown in the figure. The flow rate is* ***314.16 L/s****. It is assumed that water is an ideal fluid.*

1. Determine the pressure difference between sections: $p_2 - p_1$

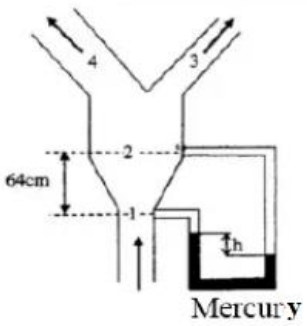
2. If a differential mercury manometer is used to measure $p_2 - p_1$, what will be the height difference h indicated by the manometer?

3.The pipe of diameter D_2 splits into two pipes of diameters D_3 and D_4

Given that the volumetric flow rates through the two branches Qv_3 and Qv_4 are equal, and that D_3=10 cm, calculate:

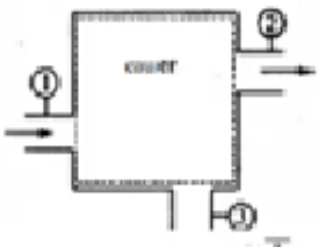
-the diameter D_4.

-and the velocity V_3.



Exersise 2 : *A vertical pipe with a diameter D=500 mm is supplied by a large reservoir with a free surface. It ends with a 90° elbow, followed by a horizontal section whose outlet is equipped with a convergent nozzle of diameter d=100 mm (see figure).At the outlet of the nozzle, water (with density*

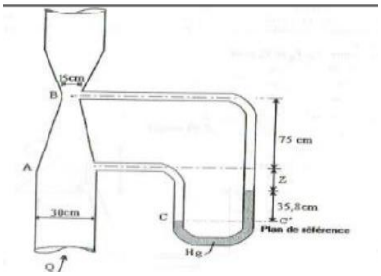
$\rho=1000\ kg/m^3$ *flows into the open air at atmospheric pressure* $Patm=100\ kPa$.



The height difference between the free surface of the reservoir and the jet axis is $H=10\ m$. *The water is considered an ideal fluid and the flow is steady.*

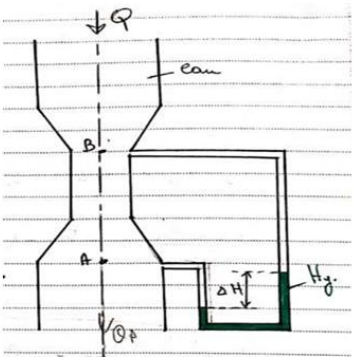
1. *Calculate the volumetric flow rate qv of the water flowing through the pipe.*
2. *Assuming the water is weightless, determine the resultant force exerted by the water on the elbow (limited between the sections with diameters D at E and ddd at F), assuming the pressure at E and F is the same (PE= PF).*

Exercise 3 : *In the Venturi tube shown in Figure 3,* ***water*** *with a density* $\rho=1000\ kg/m^3$ *is flowing. The differential manometer shows a mercury level difference of 35.8 cm.*



Calculate the water flow rate through the device, assuming no energy is lost between points A and B.

Exercise 4 : *The difference in height between the two mercury columns in a Venturi meter is*
$\Delta h=6\ cm$.



1*. What are the velocity at point A and the flow rate?*

Assume no head losses. The given data are:

$S_A=100\ cm^2$, $S_B=10\ cm^2$, $\rho Hg=13600 Kg/m^3$, $\rho water=10^3 Kg/m^3$

$Z_A=100\ cm$, $Z_B=118\ cm$

Exercise 5 : *A pipe with a cross-sectional area of 1 cm² carries water under a pressure of 90 mm Hg. The water flows with a volumetric flow rate of 5 cm³/s and a velocity vvv. At a certain point, the pipe narrows.*

1. *What is the cross-sectional area of the narrower section if the velocity of the water in that section is* ***twice*** *the velocity in the original section?*
2. *What is the* ***pressure*** *of the water in the narrowed section*

We give : S_1=1 cm^2, P1=90 mm , Qv=5 cm^3/s , ρ=1000 kg/m^3 ,Patm=1.013×10^5 Pa.

Chapiter 4 : Real fluid dynamics

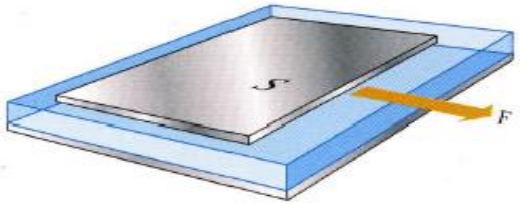
Chapiter 4 : Real fluid dynamics

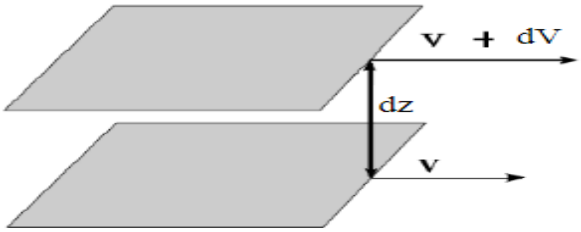
I. Introduction

In a real fluid in motion, the frictional forces play a significant role, unlike in a perfect fluid where the friction is negligible. These forces alter the course of the flow and convert part of the energy of the fluid into heat. This energy dissipation results from the friction between the adjacent fluid layers or between the fluid and the walls of the pipe. As a result, Bernoulli's theorem is not applicable under such conditions: $P + \frac{1}{2}\rho v^2 + \rho g z \neq Cst$

II. Viscosity

The viscosity of a fluid creates a frictional force F which opposes the movement. It is given by : $F = \eta . S . \frac{\Delta V}{\Delta x}$ *(1). The two layers of fluid circulate in parallel at different speeds.*





S : Friction surface..

$\frac{\Delta V}{\Delta x}$: *Grandient of speed or shear rate.*

η: *Dynamic viscosity coefficient, which translates the resistance to deformation..*

$[\eta] = ML^{-1}T^{-1}$: *Unit of the viscosity in the SI system (Pa.s). It bears the name of* ***poiseuille.***

1 Pa.s = 1Pe . In the system CGSA, the unit is named after Poise: 1Po = 0.1Pa.s=0.1Pe.

F : Frictional force exerted between two layers of fluid.

II.1 Kinematic viscosity

It reflects the ability of an agitated fluid to return to rest. It is given by the following relation : $\nu = \frac{\eta}{\rho}$.

It is expressed in m^2/s *in the system (SI) and in stockes (St) = 1st =* $10^{-4} m^2/s$.

We will find, in the table below, some values of the dynamic and kinematic viscosity of some fluids.

fluide	η *(Pa.s)*	ν (m^2/s)
water (0°C)	$1{,}787\ 10^{-3}$	1.79×10^{-6}
water (20°C)	$1{,}002\ 10^{-3}$	$1.0\ 10^{-6}$
Olive oil (20°C)	$100\ 10^{-3}$	$4.0\ 10^{-4}$
Mercury (15°C)	$1{,}60\ 10^{-3}$	1.141×10^{-6}
Glycerin	$1{,}30\ 10^{-2}$	$6{,}8\ 10^{-4}$
Alcohol	$1{,}31\ 10^{-2}$	$2{,}2\ 10^{-6}$

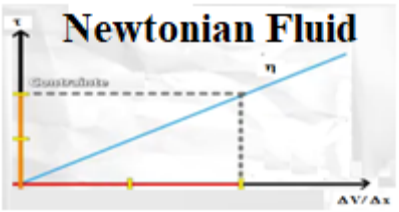
According to the equation (1), viscosity can be defined as the ratio between the shear stress $\tau = \frac{F}{S}$ *and the shear rate (* $\gamma = \frac{\Delta V}{\Delta x}$ *) , so* $\eta = \frac{\tau}{\gamma}$

II.2 Newtonian and non-Newtonian real liquids

In principle, the viscosity, which is a characteristic of the liquid, is a constant. In reality, it varies with temperature and pressure. It can also vary with the gradient of the speed. So the real fluids constitute two groups.

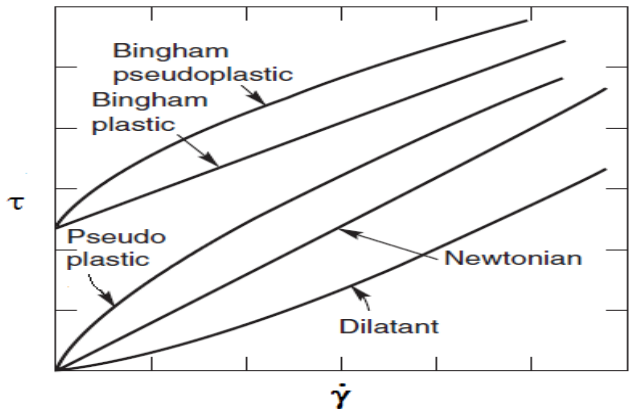
a. Newtonian fluid.

They have a constant dynamic viscosity independent of the sliding force and proportional to its deformation speed. is independent of the shear rate. given by: $\tau = \eta . \frac{\Delta V}{\Delta x}$ *where* η *is Dynamic viscosity*



b. Non-Newtonian fluid

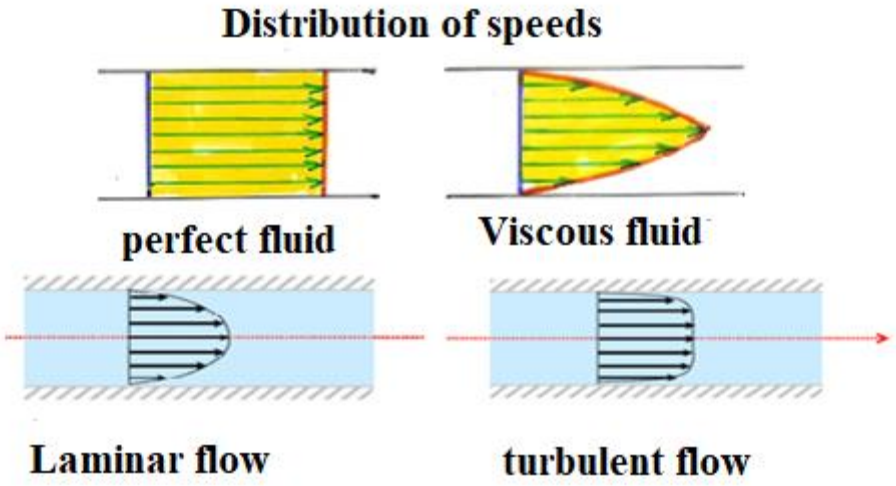
A fluid is non-Newtonian if its viscosity is a function of the speed at which an object moves inside, as well as the stresses to which it is subjected. It is the particularity of solidifying or liquefying depending on the stress subjected (Example honey). In a non-Newtonian fluid, the flow curve (shear stress as a function of the shear rate) is nonlinear or does not pass through the origin..



Flow Behavior of Newtonian and Non-Newtonian Fluids

II.3 Velocity profile in a moving fluid

The profile of the speeds, in laminar regime is parabolic. *En In turbulent conditions, the profile of the speeds, in a cylindrical pipe, is no longer parabolic; because of the turbulence. The speeds are standardized over a wide domain. A sudden variation in speed is observed in the vicinity of the walls.*



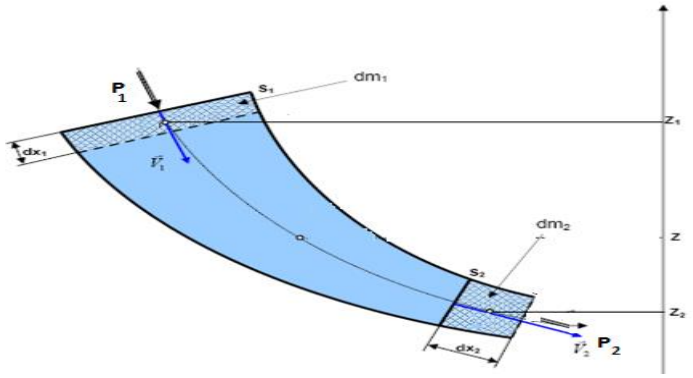
III. Real fluid

III.1 Bernoulli equation for a real fluid

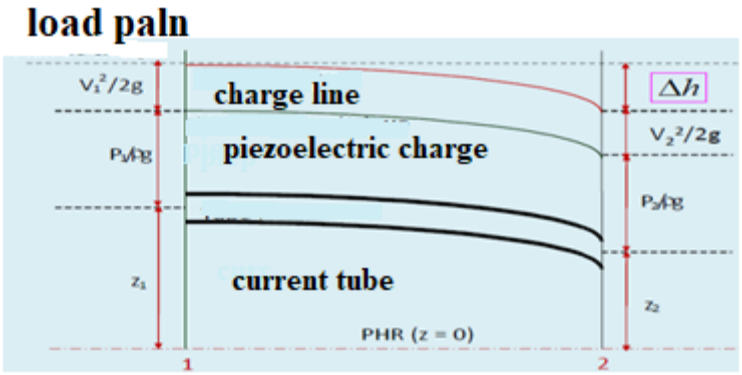
the fluid is viscous, part of the energy of the fluid dissipates, it is absorbed by the work of frictional forces. The Bernoulli equation is not verified and it becomes :

$$\frac{1}{2}\rho V_1^2 + \rho.g.Z_1 + P_1 = \frac{1}{2}\rho V_2^2 + \rho.g.Z_2 + P_2 + \Delta H_{12}$$

where ΔH_{12} is the total pressure drop between the sections 1 and 2.



Graphical Representation of Bernoulli's Theorem in Real Flow.

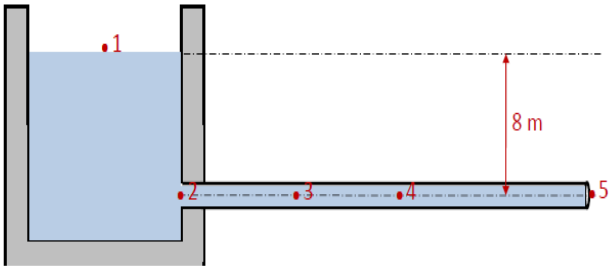


Example : *Water flows from a reservoir through a pipe ϕ 250 mm. It is assumed that the pressure drops are as follows:*

from 2 to 3 : $\Delta h_{2\text{-}3} = 1.45$ *m*

from 3 to 4 : $\Delta h_{3\text{-}4} = 1.65$ *m*

from 4 to 5 : $\Delta h_{4\text{-}5} = 1.90$*m*



1. Determine the flow rate of the pipe and the pressures at the points 3 and 4.

2. Draw the piezometric line and the load line. N.B: The pressure drop between 1 and 2 is supposedly negligible.

Solution :

Flow rate of the installation: $Q = V.S = V.\frac{\pi}{4}r^2$

To calculate V, we must apply Bernoulli's theorem between the free surface of the reservoir and the end of the pipe.

$\frac{1.}{2}\rho V_1^{\ 2} + \rho.g.Z_1 + P_1 = +\rho.g.Z_5 + P_2 + \Delta H_{15}$, $V_1 = 0$ *(large tank dimensions)*

$P_1 = P_5 = 0$ *(taken as origin)*

$$Z_1 = 5 + \frac{1}{2}\rho.V_5^{\ 2} + \Delta H_{1-5} \quad , \qquad V_5 = \sqrt{2g(Z_1 - Z_2 - \Delta H_{1-5})}$$

$$V_5 = \sqrt{2.10\big(8 - (1.45 - 1.65 + 1.9)\big)} = 7.67\ m/s$$

The flow rate is calculated by: $Q = 7.67x\frac{3.14.(0.25)^2}{4} = 0.377 m^3/s$

Calculation of the pressures at points 3 and 4

Bernoulli's theorem, between points 1 and 3: $\frac{V_1^2}{2.g} + Z_1 + \frac{P_1}{\rho.g} = \frac{V_3^2}{2.g} + Z_3 + \frac{P_3}{\rho.g} + \Delta H_{13}$

$$Z_1 + \frac{P_1}{\rho.g} = Z_3 + \frac{P_3}{\rho.g} + \Delta H_{13} \ , \ \frac{P_3}{\rho.g} = Z_1 - Z_3 - \frac{V_3^2}{2.g} - \Delta H_{13} = 8 - \frac{7.67^2}{2\ x\ 9.81} - 1.45 = 3.55.$$

In an analogous way, Bernoulli's theorem applied between 3 and:

$$\frac{V_3^2}{2.g} + Z_3 + \frac{P_3}{\rho.g} = \frac{V_4^2}{2.g} + Z_4 + \frac{P_4}{\rho.g} + \Delta H_{34} \quad , \quad V_3 = V_4 \ , Z_3 = Z_4 \quad , \quad \frac{P_3}{\rho.g} = \frac{P_4}{\rho.g} + \Delta H_{34}$$

$$\frac{P_4}{\rho.g} = \frac{P_3}{\rho.g} - \Delta H_{34} = 3.55 - 1.63 = 1.92\ m$$

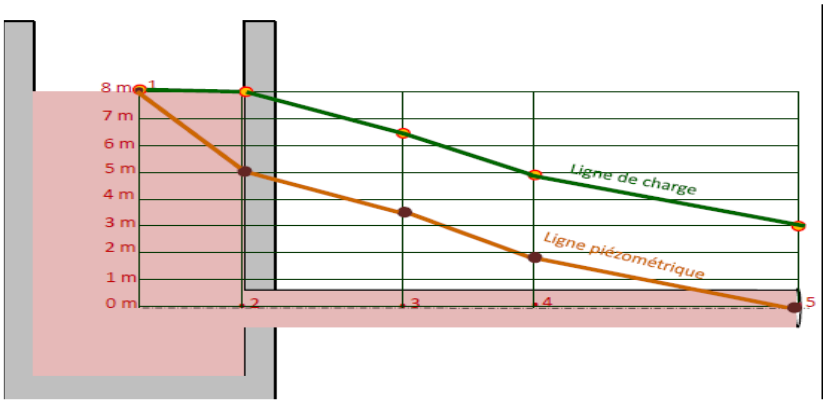
Drawing of the piezometric line and the charge line

To perform the plot of H and $\left(\frac{P}{\rho.g} + Z\right)$*, it is necessary to determine the pressure at the points 2.*

Bernoulli's theorem between points 1 and 2: $\frac{V_1^2}{2.g} + Z_1 + \frac{P_1}{\rho.g} = \frac{V_2^2}{2.g} + Z_2 + \frac{P_2}{\rho.g} + \Delta H_{12}$

$$Z_1 + \frac{P_1}{\rho.g} = \frac{V_2^2}{2.g} + Z_2 + \frac{P_2}{\rho.g} \ , \ \frac{P_2}{\rho.g} = Z_1 - Z_2 - \frac{V_2^2}{2.g} = 8 - \frac{7.67^2}{2.9.81} = 5m$$

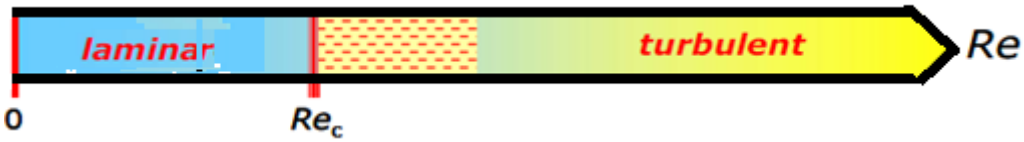
Point	Charge (m)	Piézoélectric Charge (m)
1	$\frac{V_1^2}{2.g} + Z_1 + \frac{P_1}{\rho.g} = 0 + 8 + 10 = 8$	$Z_1 + \frac{P_1}{\rho.g} = 8 + 0 = 8$
2	$H1 - \Delta H_{12} = 8$	$Z_2 + \frac{P_2}{\rho.g} = 0 + 15 = 5$
3	$H_2 - \Delta H_{23} = 8 - 1.45 = 6.55$	$Z_3 + \frac{P_3}{\rho.g} = 0 + 3.55 = 3.55$
4	$H_3 - \Delta H_{34} = 16.55 - 1.65 = 4.9$	$Z_4 + \frac{P_4}{\rho.g} = 0 + 1.92$
5	$H_4 - \Delta H_{45} = 14.9 - 1.90 = 3$	$Z_5 + \frac{P_5}{\rho.g} = 0 + 0 = 0$



III.2 Flow regimes (nature of the flow)

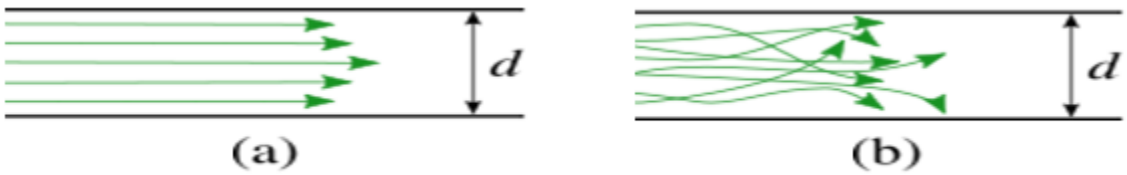
The experiments carried out by ***Reynolds en 1883*** *during the flow of a liquid in a rectilinear cylindrical pipe in which a stream of colored liquid also arrives, showed the existence of two flow regimes.*

*The flows for a real fluid are classified into 2 main regimes : Laminar and turbulent, separated by a transient phase called '****the critical regime'****.*



a- Laminar flow : *A flow is laminar if the current lines slide on top of each other remaining parallel. Laminar flow usually exists if the velocity is low.*

b- turbulent flow : *The turbulent flow is an unstable flow and its structure is chaotic.*



c- Reynolds number : *To characterize these flow regimes, a dimensionless number is introduced, called* ***Reynolds number****, given by :* $R_e = \frac{V.D}{\nu} = \frac{\rho.V.D}{\eta}$

V :	*Average flow speed*	D :	*Diameter of the circular pipe.*
ν :	*Kinematic viscosity*	η :	*dynamic viscosity*
ρ :	*the density of mass of the fluid.*		

The limits of the Reynolds number define the different flow regimes.

$$R_e \leq 2000 \Rightarrow the\ flow\ is\ laminar.$$

$$2000 \leq R_e < 4000 \Rightarrow the\ flow\ is\ \boldsymbol{critical}$$

$$R_e \geq 4000 \Rightarrow the\ flow\ is\ turbulent$$

IV. The pressure losses

La The pressure drop in a real fluid is the consequence of several parameters:

1- *The viscosity of the fluid.*

2- *The friction of the fluid on the walls of the pipe, therefore its roughness.*

3- *The Change of direction or section of the driving.*

4- *La The presence of valves, counters...*

The following pressure losses are distinguished:

IV. 1 Linear pressure drops (regular or systematic)

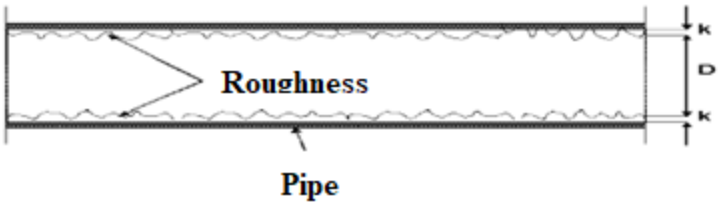
This kind of loss results from the internal friction that occurs in liquids ;

The linear pressure drops are calculated by the formula of ***Darcy-Weisbach (1857).***

Between two points separated by a length L, in a pipe of diameter D a pressure loss p appears p. expressed in the following form: $\Delta P = \lambda \frac{\rho V^2}{2} . \frac{L}{D}$

a. ***Notion of roughness of pipes:*** *Roughness is a characteristic of the surface condition of a solid material.*

Unlike a smooth surface, a rough surface implies a surface condition whose irregularities have a direct action on the frictional forces. A rough surface can be considered as consisting of a series of elementary protuberances characterized by a height, denoted k and called «Roughness». In order to compare the roughness with respect to the diameter of the pipe, the ratio is introducedt : ε = k/D (Relative roughness). K is the absolute roughness.

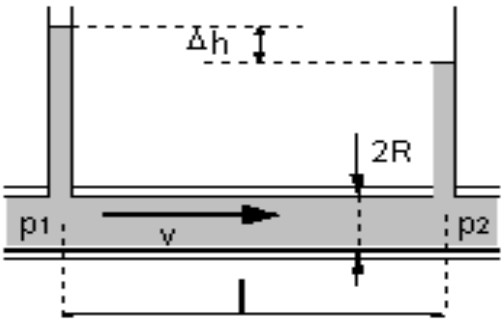


Several formulas are proposed for the calculation of which depends on the flow regime.

IV.1.1 Case of laminar flow (Re < 2000) : *In this case it can be shown that the coefficient is only a function of the Reynolds number Re. The state of the surface does not intervene :* $\lambda = \frac{64}{R_e}$.

IV.1.2 Poiseuille's Law: *For a laminar flow, in a horizontal cylindrical pipe, the flow-volume of a fluid is given by:* $Q_v = \frac{\pi R^4}{8\eta L}(P_1 - P_2)$.

1- Q_v *: flow-volume* ($m^3 \cdot s^{-1}$),
2- R *: inner radius (m),*
3- η *: dynamic viscosity of the fluid (Pa·s)*
4- L *: length between points (1) and (2) (m),*
5- P_1 *and* P_2 *: fluid pressures at the points (1) and (2) (Pa)*

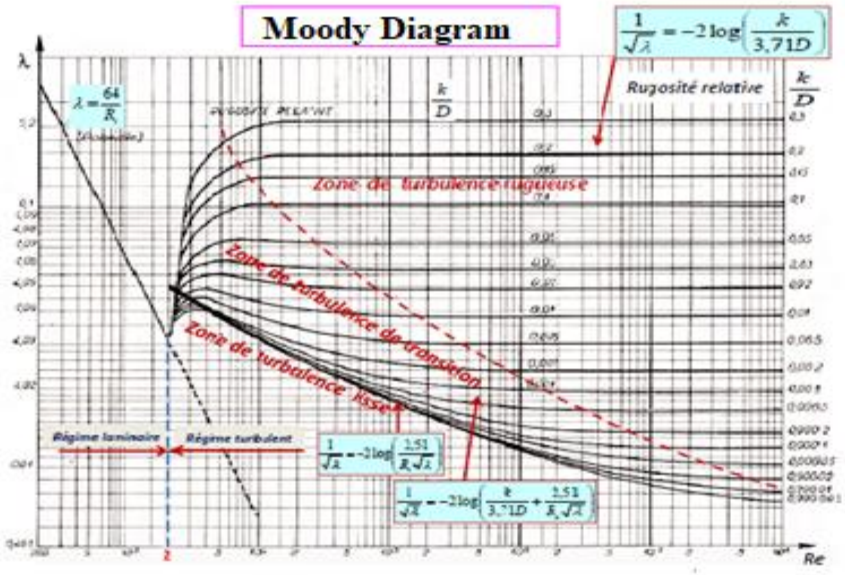


IV.1.3 Case of turbulent flow (Re > 3000) : *In turbulent conditions, the state of the surface becomes sensitive and its influence is all the greater as the Reynolds number Re is big. All the works have shown the influence of the roughness and we have subsequently focused on looking for the variations of the coefficient as a function of the Reynolds number Re and the roughness ε of the pipe. The formula of* **Colebrook** *is currently considered as the one that best translates the phenomena of flow in turbulent regime. It is presented in the following form :* $\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right)$

We often use simpler empirical formulas valid for particular cases and in a certain domain of the Reynolds number, for example:

Formula of Blasius : *(for* smooth pipes*and Re < 10^5) :* $\lambda = \frac{0.316}{R_e{}^{0.25}}$

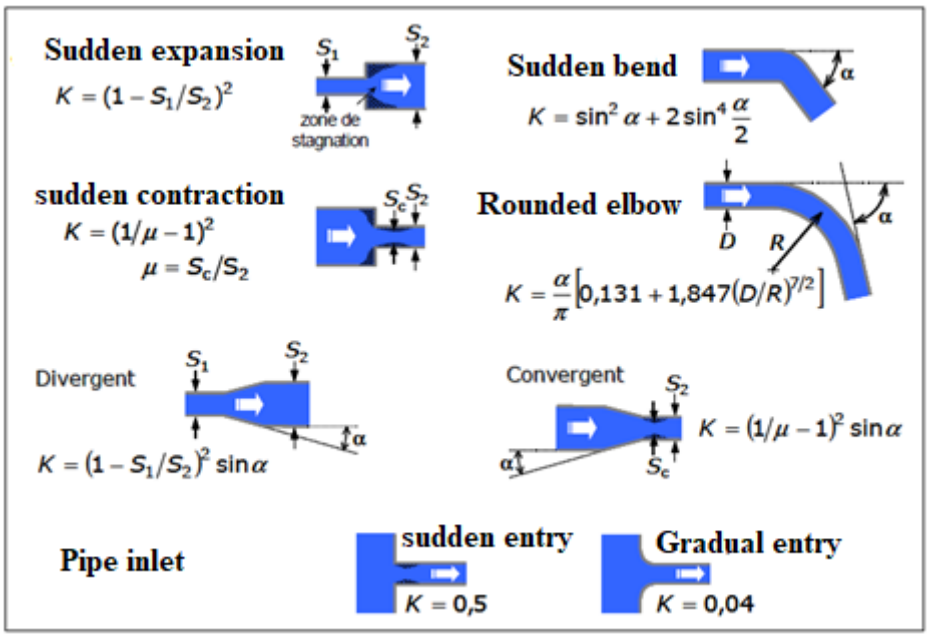
Moody's Diagram: *Nikuradse's work on pressure losses in pipes made it possible to develop a graph (Moody's Diagram) making it possible to determine the coefficient λ as a function of Re and for the different types of flow and relative roughnesses k/D.*



IV.2. The singular pressure drops.

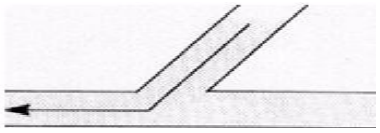
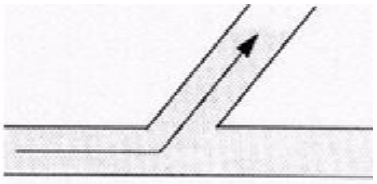
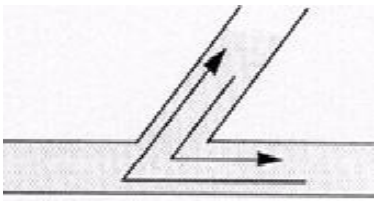
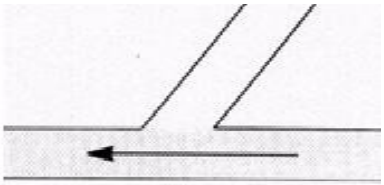
The singular pressure losses result from accidents along the fluid path, the presence of elbow, connection, etc., the change of direction and section. This pressure drop is calculated by : $\Delta P = k\frac{\rho V^2}{2}$

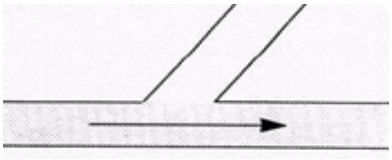
K *is called singular (dimensionless) pressure drop coefficient. This coefficient depends on the geometry and the Reynolds number. It is given in the "dictionaries of pressure drops". Some typical singularities are reproduced below.*



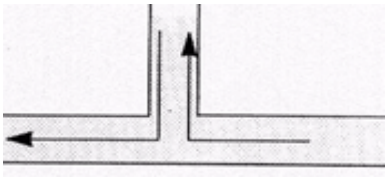
SINGULARITY	*LOSS OF charge*	*VALUE OF K*

Connections and branches of the same diameters (according to SCIMEMI)

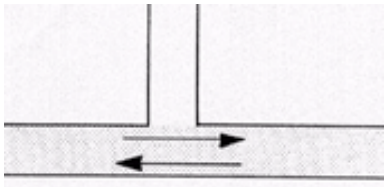
	$KV^2_m / 2g$	$K = 0.50$
	$KV^2_m / 2g$	$K = 1.00$
	$KV^2_m / 2g$	$K = 3.00$
	$KV^2_m / 2g$	$K = 0.05$



$KV^2_m / 2g$ $K = 0.15$



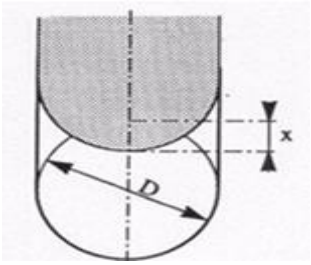
$KV^2_m / 2g$ $K = 1.50$



$KV^2_m / 2g$ $K = 0.10$

SINGULARITY	LOSS OF Charge TAPWARE	VALUE OF K

10 **Shut-off valve**

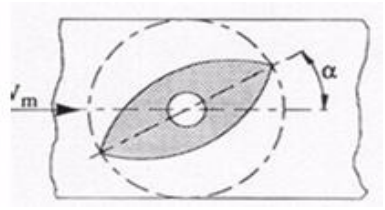


$KV^2_m / 2g$

x/D	1/8	2/8	3/8	4/8	5/8	6/8	7/8
K	0,07	0,26	0,81	2,06	5,52	17	98

The flow rate decreases noticeably at the end of the movement.

11 **Butterfly valve**

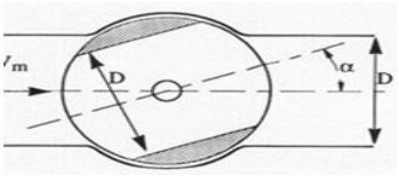


$KV^2_m / 2g$

α°	5	10	15	20	30
K	0,24	0,52	0,90	1,5	3,9

α°	40	45	50	60	70
K	11	19	33	118	750

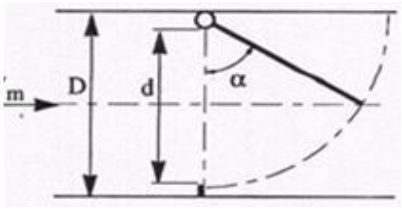
12 **Plug valve** $KV^2_m / 2g$



α°	5	10	15	20	25	30
K	0,05	0,29	0,75	1,6	3,1	5,5

35	40	45	50	55	60	65
9,7	17	31	53	110	206	490

Batton flap valve

13 

$KV^2_m / 2g$

D = 0.73

α°	15	20	25	30	35
K	90	62	42	30	20

α	40	45	50	60	70
K	14	9,5	6,6	3,2	1,7

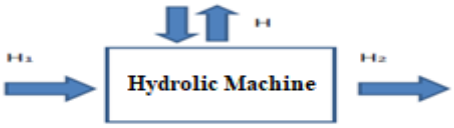
The company PONT A MOUSSON gives the values of K for the valve positioned horizontally, as in the diagram, while M. CARLIER (General Hydraulics applied) gives the same values for the valve positioned vertically.

IV.3. The total pressure drop

During a flow in a hydraulic pipe, the total pressure losses are the addition of two types of pressure losses (regular and singular).

$$\Delta P_{total} = k\frac{\rho V^2}{2} + \Delta P = \lambda\frac{\rho V^2}{2}.\frac{L}{D}$$

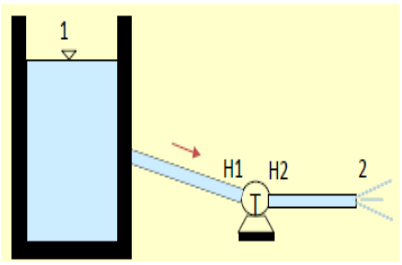
V. ***Bernoulli's theorem with energy transfer:*** *In industrial embodiments, it is common for a hydromechanical device, placed in a fluid stream, to allow a transformation of mechanical energy into hydraulic energy (a pump for example) or vice versa (a turbine).*

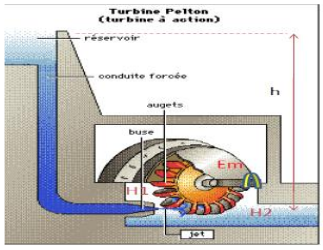


if Ht energy is supplied by the flow to a mechanical device, the Bernoulli equation becomes:

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 - H_t = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + \Delta h$$

Receiving machines : $H_2 < H_1$

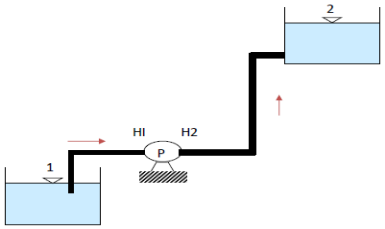




*If **Hp** energy is supplied to the flow by means of a pump, the Bernoulli equation becomes:*

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 + H_p = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + \Delta h$$

Lifting machines (positive displacement pumps): $H_2 > H_1$



V.1 Powers of the flows :

*If a flow of a fluid takes place at a rate **Q (m^3/s)**, with a specific weight **ρg (N/m^3)** and total energy **H (m)** $\Rightarrow$ Power of the flow **(W)***

$$P = \rho.g.Q.H \ (w)$$

*If a machine supplies energy to the fluid **Hp** $\Rightarrow$ Hydraulic power of the machine **(W)***

$$P = \rho.g.Q.H_p \ (w) \ , \ H_p \ (m)$$

*If a machine consumes energy from the fluid **Ht** $\Rightarrow$ Hydraulic power of the machine*

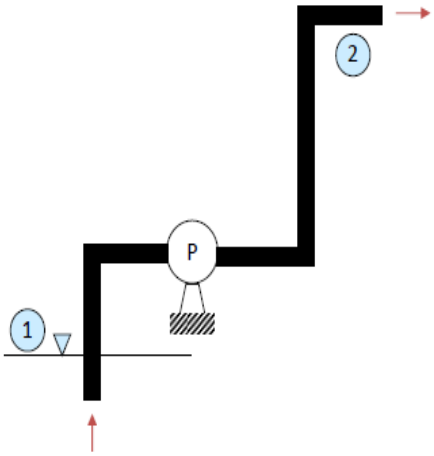
$$P = \rho.g.Q.H_t \ (w) \ , \ H_t \ (m)$$

Example : *Either a pumping circuit delivering 0,024 m^3/s of water. The pipe has a diameter of 160 mm and opens out into the open air, to 6 m above the body of water. In the case of a perfect liquid, calculate the load height and the hydraulic power supplied by the pump.*

Solution : *The useful power of the pump is expressed by the relationship* $P = Qv.\ H_p$

$$P_1 + \frac{1}{2}\rho_{eau}v^2{}_1 + \rho_{eau}gz_1 + H_p = P_2 + \frac{1}{2}\rho_{eau}v^2{}_2 + \rho_{eau}gz_2$$

Points 1 and 2 are subject to atmospheric pressure , $P_1 = P_2 = P_{atm}$. *Point 1 is located on a free surface (large tank)*$v_1 \sim 0$.



where $H_p = \frac{1}{2}\rho_{eau}v^2{}_2 + \rho_{eau}g(z_2 - z_1)$. *Let's calculate the speed* v_2. $V = Q_2/S_2 = 4\ x0{,}024/\pi x\ 0{,}16^2 = 1{,}19$ *m/s.*

d'ou $H_p = 10^3\left(\frac{1{,}19^2}{2} + 9{,}81x6\right) = 59568{,}05Pa$. *Enfin* $P = 59568{,}05x0{,}024 = 1{,}43K\ w$

VI-1 Exercises

Exercise 1 : *Determine the flow regime in a pipe with a diameter of 3 cm for:*

1. *Water (with a kinematic viscosity of* $1.10^{-6}\ m^2/s$*) circulating at the speed v = 10.5 m/s.*

2. *Heavy fuel oil at 50 ° C (kinematic viscosity* $110.10^{-6}\ m^2/s$*) circulating at the same speed.*

3. *Heavy fuel oil at 10 ° C (kinematic viscosity 290.10 - 6 m2 /s) circulating at the same speed.*

Exercice 2 : *Heavy fuel oil with a dynamic viscosity* $\mu = 0{,}11$ *and a density* $d = 0{,}932$ *circulates in a pipe with a length L = 1650 m et de diamètre D=25 at a volume flow rate qv = 19,7 l/s. The density of masss of the water* $\rho . = 1000\ Kg/m^3$.

1. *Determine the kinematic viscosity v of the fuel oil..*

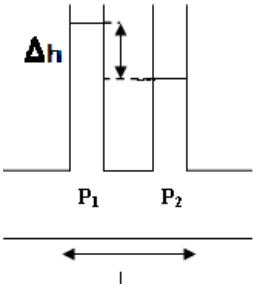
2. *Calculate the flow velocity V.*

3. *Calculate the Reynolds number Re.*

4. Deduce the nature of the flow.

5. *Calculate the pressure drop in the pipe.*

Exercice 3 : *To measure the viscosity of an oil, the device shown schematically in the figure is used. The oil is poured into a horizontal tube with a diameter of 7.0 mm and comprising two vertical manometric tubes located at L = 600 mm from each other. The flow-volume of this flow is adjusted to. The flow-volume of this flow is adjusted to* $4{,}0\times10^{-6}\ m^3/s$.



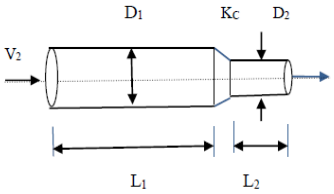
The difference in level of the oil between these two tubes is then Δh = *267 mm. The density of mass of the oil is* $910 kg/m^3$.

1. *Calculate the pressure difference* P_1-P_2

2. *Calculate the dynamic viscosity of the oil.*

3. *Determine the nature of the flow.*

Exercice 4 : *A pipe consisting of two sections (fig.3) of different diameters and lengths transporting an oil of density of mass* ρ = *850 kg/m3 and kinematic viscosity* $10^{-5}\ m^2/s$. *Calculate the pressure difference between the inlet and the outlet of the pipe knowing that:* L_1 = *5 m ,* L_2 = *0. 2m,* D_1 = *20 mm,* D_2 = *5 mm,* V_1 = *0.5 m/s et* K_c = *0.*



Exercice 5 : *Viscosity oil* μ = *0.11 Pa.s and density 0.9 circulates in a pipe of length 1650 m et de diameter 25 cm at a volume flow rate 19.7 l /s.*

1. *Determine the kinematic viscosity of oil in the SI system and the CGS system*

2. *Calculate the flow velocity and the mass flow rate.*

3. *Calculate the Reynolds number and deduce the nature of the flow.*

4. *Determine the linear pressure drop coefficient and calculate the loss in the pipe.*

Exercice 6 : *A* ***horizontal***, ***smooth*** *pipe of* ***inner diameter*** $D=80\ mm$ *and* ***length*** $L=15\ m$ *carries* ***water*** *at a temperature of 20∘C with a* ***volumetric flow rate*** *of* $Q=2.0\ L/s$.

Given:

- *Water density:* $\rho=1000\ kg/m^3$
- *Kinematic viscosity of water:* $v=1.0\times10-6\ m^2/s$
- *Blasius formula (for smooth pipes and Re<105Re):*
  $\lambda=0.3164/Re^{0.25}$

1.Calculate the average velocity of the fluid in the pipe.

2.Calculate the Reynolds number and determine the flow regime (laminar or turbulent).

3.Using the Blasius formula, calculate the friction factor.

4.Calculate the linear pressure loss ΔP along the pipe.

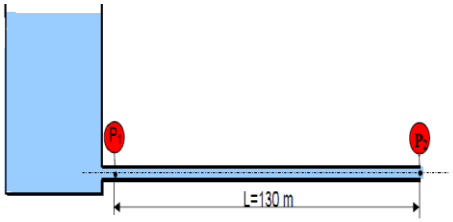
Exercice 7 : *A fluid with a density of 961 kg /m3 flows at a speed of 1.5 m/s in a horizontal pipe with a diameter of d = 120 mm from a large section tank open to the open air. Two separate 130 m pressure gauges are installed on the horizontal part of the tube. A pressure drop of 1.5bar is recorded between the two pressure intakes.*

1. *By applying the Bernoulli equation, determine the linear pressure losses ΔP.*

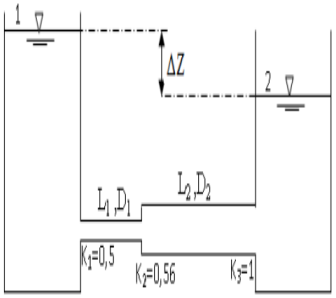
2. *Determine the linear pressure drop coefficient λ as a function of ΔP, ρ, L, d and V.*

3. *We assume the laminar flow, calculate the Reynolds number as a function of λ.*

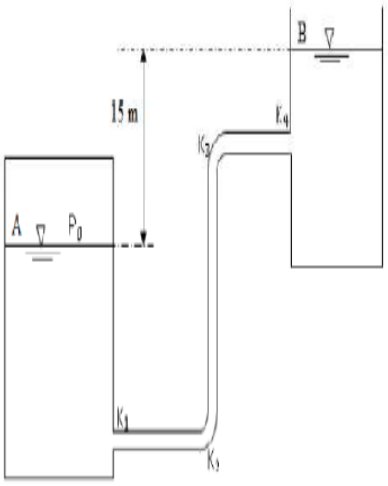
4. Deduce the kinematic viscosity of the fluid.



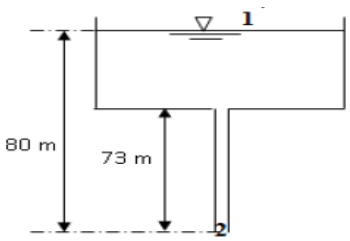
Exercice 8 : *Two tanks are connected by two pipes, placed in series with a diameter d_1 = 15 cm et d_2 = 30 cm et de longueur L_1= 50 m et L_2=160 m. he roughness of the two pipes is 0.1 mm and the flow rate of the flow is 0,1 mm and the flow rate of the flow is 0,1 m^3/s. Determine the difference in the level of the free surface between the two tanks (Δz). (ρ=1000kg/m^3, v=1.00510-6 m^2/s)*



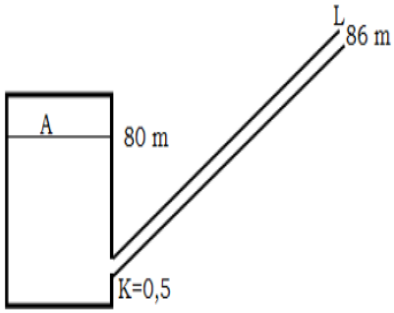
Exercise 9 : *thanks to an over pressure P_0, the water circulates from the tank T to the reservoir s by means of a pipe with a diameter d = 300 mm ε = 0,3 mm in roughness and a length l = 170 m. The coefficients of the singular pressure drops are K_1 = 0,5 at the outlet of the tank T, K_2 = K_3 = 0,15 for the two bends and K_4. = 1 at the inlet of the tank B. Determine the manometric pressure Q_v =200 l /s. we give: $\rho = 10^3$ kg /m^3, g = 9,81 m/s^2 et ν=1,005. 10^{-6} m^2/s.*



Exercise 10 : *A water tank is drained by a vertical pipe. The diameter of the pipe is 50 mm and its length is 73 m. Neglecting the singular pressure drops and knowing that the linear pressure drop coefficient is λ = 0.025. Calculate the flow rate.*



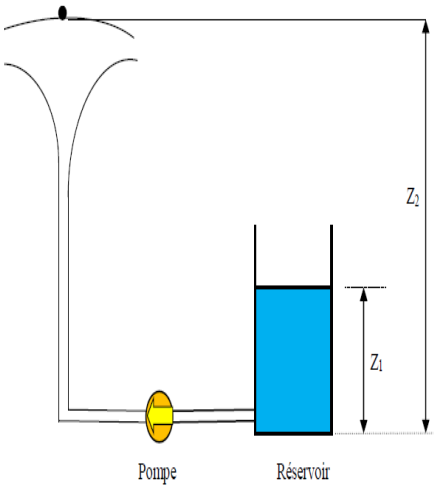
Exercise 11 : *De l'huile circule du réservoir A a travers 150 m de tuyau neuf de fonte asphaltée de 150 mm de diamètre jusqu'au point B de cote 30 m, comme le montre la Figure ci-dessous. Quelle devrait être la pression en A pour que le débit de l'huile soit de 13 l / s ? On donne : Densité de l'huile 0.84, et ν = 2.1 10^{-6} m^2 / s. Utiliser ε = 0.12 mm.*



Exercice 12 : *A jet of water is fed from a large tank by means of a centrifugal pump with a volume flow rate of 2 l/s, through a pipe with a length of 15 m and an internal diameter of 3 cm. The pipe has an elbow of 90° (Kc = 0.3), ρ eau = 10^{-3} Pa.s.*

1. *Calculate the speed of the flow.*

2. *Calculate the Reynolds number and specify the nature of the flow.*



VI .2 Additional exercises

Exercise 1 : Calculation of the dynamic viscosity: *A fluid has a dynamic viscosity of 0.003 Pa·s 0.003 Pa·s and a density of 900 kg/m3. Calculate the kinematic viscosity of the fluid.*

Exercise 2 : Viscous flow in a tube: *Consider a viscous fluid that flows through a cylindrical tube with a radius r = 0.02m at a speed V = 0.1m / s. The dynamic viscosity of the fluid is μ=0.005Pa·s. Calculate the Reynolds number for this flow, then determine the nature of the flow.*

Exercise 3 : Poiseuille equation: *Use Poiseuille's equation to calculate the water flow rate (ρ = 1000 kg/m3) through a cylindrical tube of length L = 2 m and radius r = 0.01 m. The pressure difference between the ends of the tube is ΔP = 200 kPa. The dynamic viscosity of the water is μ=0.001Pa·s.*

Exercise 4: Écoulement visqueux dans une conduite : *Viscous flow in a pipe, the length of the pipe is L=10 m. It is assumed that the density of mass of the fluid is ρ=1000 kg / m3, and that the pipe is smooth. 1- Determine the Reynolds number and conclude on the flow regime (laminar or turbulent). 2- Assuming a turbulent flow, estimate the value of the coefficient of friction using the empirical formula de Blasius*

Exercise 5 : *Calculer en utilisant le diagramme de Moody, la perte par frottement dans une conduite en acier soudé d'un diamètre de 900mm sur une longueur de 500m parcouru par débit de 2.3 m^3/s. la viscosité cinématique est 1.31 10^{-6} m^2/s. ε = 0.6mm.*

Exercises Solutions

Exercises Solutions

Solution to Exercise 1

$P_1 = P_3$ (1)

$P_1 - P_4 = \rho_{eau} . g . h_1$

$P_4 = P_A$

$P_1 = \rho_{eau} . g . h_1 + P4 \Rightarrow P_1 -= \rho_{eau} . g . h_1 + P_A$ **(2)**

$P_3 - P_2 = \rho_{Hg} . g.h_2$

$P_3 = \rho_{Hg} . g.h_2 + P_2$

$P_2 = Patm$

$P_3 = \rho_{Hg} . g.h_2 + P_{atm}$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain

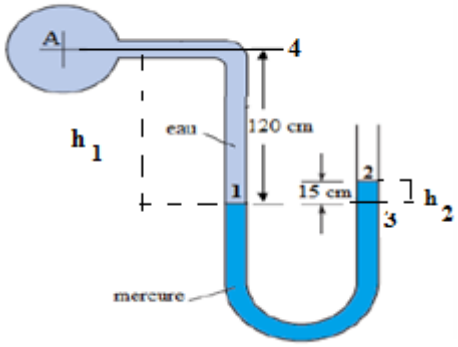
$\rho eau . g . h_1 + P_A = \rho Hg . g.h_2 + Patm$

$P_A = \rho Hg . g.h_2 - \rho eau . g . h_1 + Patm$

A.N:

$h_1 = 120$ cm , $h_2 = 5$ cm , $\rho eau = 10^3$ kg /cm^3, ρ_{Hg} = 13600 kg/ cm^3 ,Patm= 1.013 . 10^5Pa

$P_A = 1201$kPa



Solution to Exercise 2

$P_2 = P_4$ (1)

$P_2 - P_1 = \rho_{ess} . g . h_1$

$P_1 = P_{atm}$

$P_2 = \rho_{ess} . g . h_1 + Patm$ **(2)**

$P_4 - P_3 = \rho_{Hg} . g.h_2$

$P_4 = \rho_{Hg} . g.h_2 + P_3$

$P_3 = Pgaz$

$P_4 = \rho_{Hg} . g.h_2 + Pgaz$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain

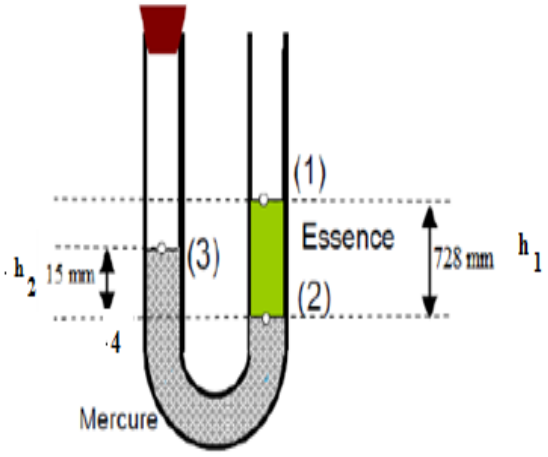
$\rho_{ess} . g . h_1 + P_{atm} = \rho_{Hg} . g.h_2 + Pgaz$

$Pgaz = \rho ess . g.h_1 - \rho Hg . g . h_2 + Patm$

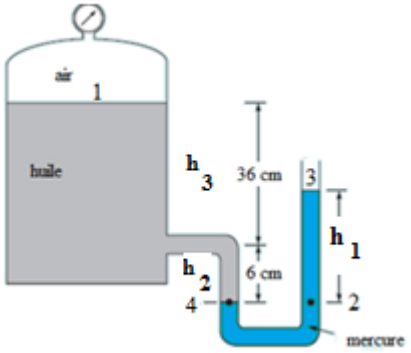
A.N:

$h_1 = 728$ mm , $h_2 = 15$ mm , $\rho ess = 700$ kg /cm^3, ρ_{Hg} = 13600 kg/ cm^3 ,Patm= 1.013 . 10^5Pa

$Pgaz = 1.044.10^5$ Pa



Solution to Exercise 3



$P_2 = P_4$ (1)

$P_2 - P_3 = \rho_{Hg} . g . h_1$

$P_3 = P_{atm}$

$P_2 = \rho_{Hg} . g.h_1 + P_{atm}$ **(2**

$P_4 = \rho_h . g.(h_2+h_3) + Pair$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obain

$\rho_{Hg} . g. h_1 + P_{atm} = \rho_H . g.(h_2+h_3) + Pair$

$h_1 = (\rho H. g.(.h_2+h_3) + P_{air} - Patm) / (\rho_{Hg} . g)$

A.N:

$h_2 = 6$ cm , $h_3 = 36$ cm , $\rho_H = 860$ kg /cm^3,

$\rho_{Hg} = 13600$ kg/ cm^3 ,Patm= 1.013 . 10^5 Pa,

$P_{air} = 100$ kP.

$h_1 = 1.7$cm

Solution to Exercise 4

1-

$P_1 = P_2$ (1)

$P_1 - P_3 = \rho_1 . g. h_3$

$P_1 = \rho_1 . g. h_3 + P3$

$P_3 – P_5 = \rho_2 . g.h_1$

$P_3 = \rho_{Hg} . g.h_2 + P_5$

$P_5 = Patm$

$P_1 = \rho_1 . g.h_3 + \rho_2 . g.h_1 + Patm$ **(2)**

$P_2 - P_4 = \rho_3 . g.h_2$

$P_2 = \rho_3 . g.h_2 + P_4$

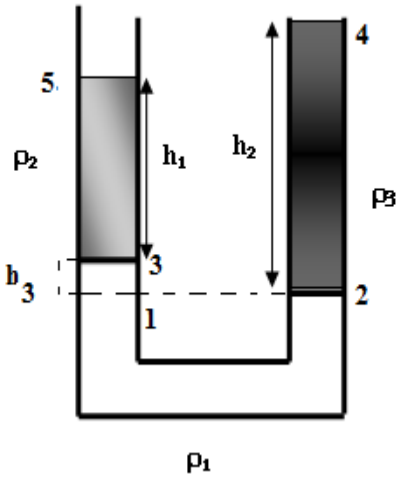
$P_4 = Patm$

$P_2 = \rho_3 . g.h_2 + P_{atm}$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain

$\rho_1 . g.h_3 + \rho_2 . g.h_1 + Patm = \rho_3 . g.h_2 + P_{atm}$

$h_3 = (\rho_3.h_2 - \rho_2 . h_1) / \rho_1$



A.N:

$\rho_1 = 13600$ kg/ cm^3, $\rho_2 = 1000$ kg /cm^3, $\rho_3 = 880$ kg/ cm^3

$h_1 = ?$ h2 = ?

$V_2 = h_1.S$

$S = 3,14 . (d/2)^2$

$S = 3,14$ cm^2

$h_1 = 100/3.14 = 31.84$ cm

$h_2 = V_3/S = 200/3.14 = 63.69$ cm

$h_3 = 1.78$ cm

2-In the branch L_2

3-$P_1 = P_2$ **(1)**

$P_1 - P_3 = \rho_2 . g . h_1$

$P_1 = \rho_1 . g . h_1 + P3$

$P_3 - P_5 = \rho_4 . g . h_4$

$P_3 = \rho_4 . g . h_4 + P_5$

$P_5 = Patm$

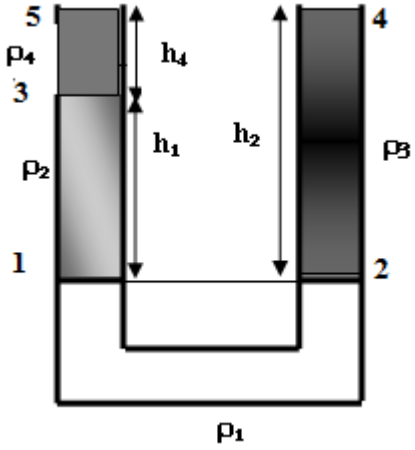
$P_1 = \rho_2 . g . h_1 + \rho_4 . g . h_4 + Patm$ **(2)**

$P_2 - P_4 = \rho_3 . g . h_2$

$P_2 = \rho_3 . g . h_2 + P_4$

$P_4 = Patm$

$P_2 = \rho_3 . g . h_2 + P_{atm}$ **(3)**



According to the equations **(1)** , **(2)** et **(3),** we obtain

$\rho_2 . g . h_2 + \rho_4 . g . h_4 + Patm = \rho_3 . g . h_2 + P_{atm}$

$h_4 = (\rho_3 . h_2 - \rho_2 . h_1) / \rho_4$

A.N. $h_4 = 34.58$ cm $\Rightarrow$

$V_4 = S . h_4 = 108.58\ cm^3$

Solution to Exercise 5

1-La pression du gaz

By applying the fundamental principle of hydrostatics:

$P_1 = P_2$ **(1)**

$P_2 - P_5 = \rho_{Hg} . g . d$

$P_2 = \rho_{Hg} . g . d + P5$

$P_5 = Patm$

$P_2 = \rho_{Hg} . g . d + Patm$ **(2)**

$P_1 = Pgaz$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain

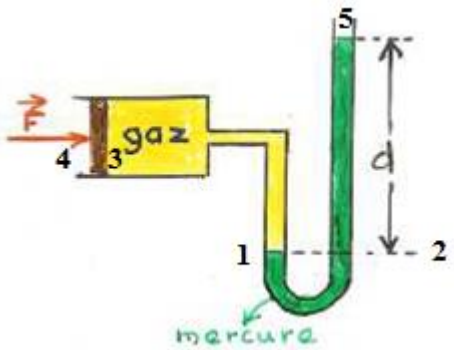
$\rho_{Hg} . g . d + Patm = Pgaz$

A.N. d= 47 cm , Patm = $1{,}013\ 10^5$ Pa ,

$\rho_{Hg} = 13600\ Kg / m^3$

$Pgaz = \rho_{Hg} . g . d + Patm = 1.6522 . 10^5$ Pa

$Pgaz_{relative} = Pgaz - Patm = \rho_{Hg} . g . d =$ 63920 Pa



2- $P_3 = P_4$

$Pgaz = F/S + Patm$

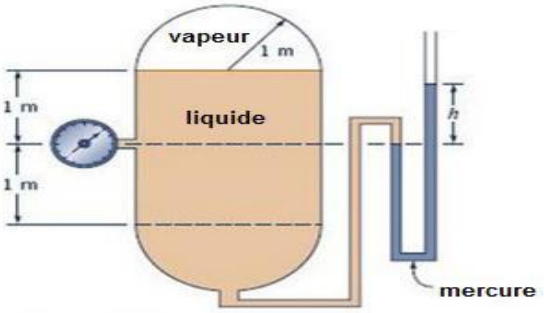
$F = (Pgaz - Patm) . S = Pgaz_{relative} . S$

A.N

$S = 20\ cm^2$

$F = 127.84$ N

Solution to Exercise 6



1. The pressure displayed by the pressure gauge

$P_{mano} = \rho_L .g\ h + P\ vapeur - Patm$
$P_{mano} = 800.10.1 + 120.10^3 - 1.013 . 10^5$
$P_{mano} = 26.7\ KPa$

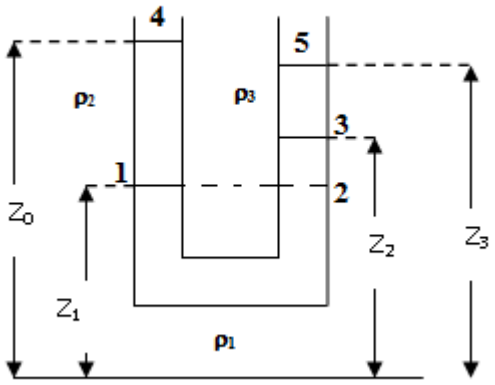
The mercury gradient h
$P_{mano} = \rho_{Hg}. g.h \Rightarrow h = P_{mano} / \rho_{Hg} . g$
$h = 0.196\ m$
.

Solution to Exercise 7

By applying the fundamental principle of hydrostatics:

$P_1 = P_2$ (1)

$P_2 - P_3 = \rho_1 . g. (Z_2 - Z_1)$
$P_2 = \rho_1 . g. (Z_2 - Z_1) + P_3$
$P_3 - P_5 = \rho_3. g.(Z_3 - Z_2)$
$P_3 = \rho_3. g. .(Z_3 - Z_2) + P_5$
$P_5 = P_{atm}$
$P_1 = \rho_1. g. (Z_2 - Z_1) + \rho_3. g.(Z_3 - Z_2) + P_{atm}$ **(2)**
$P_2 - P_4 = \rho_2. g. (Z_0 - Z_2)$
$P_2 = \rho_2. g. (Z_0 - Z_2) + P_4$
$P_4 = P_{atm}$
$P_2 = \rho_2. g. (Z_0 - Z_2) + P_{atm}$ **(3)**



According to the equations **(1)** , **(2)** et **(3),** we obtain

$\rho_1. g. (Z_2 - Z_1) + \rho_3. g.(Z_3 - Z_2) + P_{atm} = \rho_2. g. (Z_0 - Z_2) + P_{atm}$
$Z_0 - Z_1 = 0.2\ m,\ Z_1 + Z_2 = 1m,\ Z_3 - Z_2 = 0.1m$
$\rho_1. (2. Z_2 - 1) + \rho_3.0.1 = \rho_2.0.2$
$2.Z_2 . \rho_1. - \rho_1 + \rho_3.0.1 = \rho_2.0.2$

$2.Z_2 . \rho_1. = \rho_2.0.2 + \rho_1 - \rho_3.0.1$
$Z_2 = (\rho_2.0.2 + \rho_1 - \rho_3.0.1) / 2. \rho_1$
$Z_2 = 0.5\ m$
$Z_1 = 0.5\ m$
$Z_1 = 0.7\ m$
$Z_3 = 0.6\ m$

Solution to Exercise 8

I-

$P_1 = P_2$ (1)

$P_1 - P_3 = \rho_h . g . h_1$

$P_1 = \rho_h . g . h_1 + P_3$

$P_3 = Patm$

$P_1 = \rho_h . g . h_1 + P_{atm}$ **(2)**

$P_2 - P_4 = \rho_{eau} . g . h$

$P_2 = \rho_{eau} . g . h + P_4$

$P_4 = Patm$

$P_2 = \rho_{eau} . g . h + P_{atm}$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain:

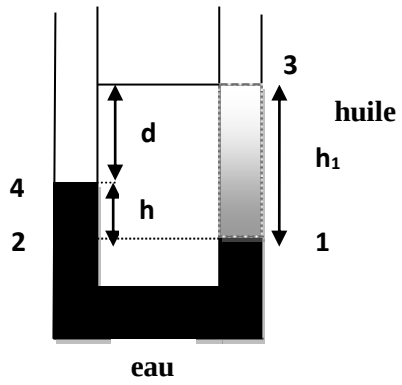
$\rho_h . g . h_1 + P_{atm} = \rho_{eau} . g . h + P_{atm}$

$h = \rho_h . h_1 / \rho_{eau}$

$h_1 = V/S = 6$ cm.

$h = 5.4$ cm

$h_1 = h + d \Rightarrow d = h_1 - h = 0.6$ cm



II-

$P_1 = P_2$ (1)

$P_1 - P_3 = \rho_h . g . h_1 + \rho_{eau} . g . h'$

$P_1 = \rho_h . g . h_1 + \rho_{eau} . g . h' + P_3$

$P_3 = Patm$

$P_1 = \rho_h . g . h_1 + \rho_{eau} . g . h' + P_{atm}$ **(2)**

$P_2 - P_4 = \rho_a . g . h_2$

$P_2 = \rho_a . g . h_2 + P_4$

$P_4 = Patm$

$P_2 = \rho_a . g . h_2 + P_{atm}$ **(3)**

According to the equations **(1)** , **(2)** et **(3),** we obtain :

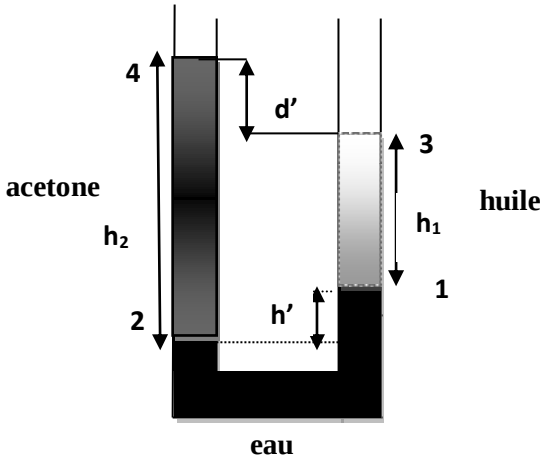
$\rho_h . g . h_1 + \rho_{eau} . g . h' + P_{atm} = \rho_a . g . h_2 + P_{atm}$

$h' = \rho_a . h_2 - \rho_h . h_1 / \rho_{eau}$

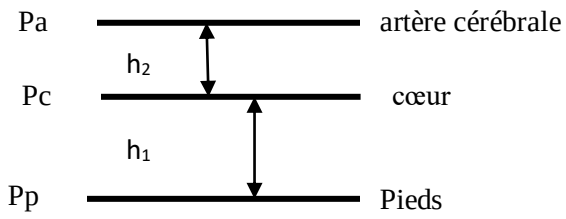
$h_2 = V'/S = 10$ cm.

$h' = 2.5$ cm

$h_2 = h + h_1 + d' \Rightarrow d' = h_2 - (h + d') = 1.5$ cm



Solution to Exercise 9



1-P_p-P_c = ρ_s . g. h_1

P_p = ρ_s . g. h_1 + P_c

1mmHg= 133.3 Pa

100 mmHg = 13330 Pa

P_p = 1200.10.120 .10^{-2} + 13330

P_p =27730 Pa

2-P_c-P_a = ρ_s . g. h_2

P_a = P_c - ρ_s . g. h_2

P_p = 13330-1200.10.60 .10^{-2}

P_p =6130 Pa

Solution to Exercise 10

1- The surface area AB

F= ρ.g.h_G.S

h_G = 4+3 = 7 m et S = 18 m^2

F= 1000. 10.7.18 =1.26 MN

The surface area AB

F= ρ.g.h_G.S

h_G = 4+3sin25° = 5.26 m et S = 18 m^2

so F= 1000.10.5.26 . 18 =928.81 kN.

2- Centre of thrust on AB

y_p = h_p = h_G + I_{XG}/h_G.S = 7+ 36^3/12/7.18 = 7.43m along the O_1 axis.

Centre of thrust on CD

yp =y_G +(I_{xG}/y_G.S)

y_G = h_G/ sin 25°= 12.46m

y_p = 12.46+ 54/12.46.18 =12.7 m suivant l'axe O_2.

3- The surface of a CD is circular.

F= ρ.g.h_G.S

where

h_G=4+R sin25° et S=π.R^2

so

F= ρg(4+0.422 R).π.R^2⇒4R^2+0.422R^3= F/ρgπ = (928, 81. $10^{3)}$ /(10^3 9.81 .3.14)=30.15

Finally, we find

$R^3+9.47\ R^2-71.44 = 0$

$R = 2.45$ m

So the diameter of the CD door is D = 4.9 m.

4- $y_p= y_G +(Ix_G/(y_G.\ S)) = (h_G/\sin 25°) + (\pi R^2/4)/11.92\ \pi R^2= 11.92 + (2.45^2/4.11.92)$

so : $y_p =$ 12.05 m along the O_2 axis.

Solution to Exercise 11

Hydrostatic force and its point of application

$F= \rho.g.h_G.\ S$

$h_G= 8-2\sin 25° =7.15$ m

$F= 10^3.9.81.7.15$

$y_p = y_G+(I_{xG}/y_G.S)$

with

$y_G=(\ h_G\ /\sin 25°) = 16.91$ m et $I_{xG} = 3.\ 4^3/12 = 16\ m^4$

$y_p= 16.91 +(16/16.91\ x12) = 16.988$ m

2- Force R required to open the door.

The force R acting on A to open the door can be deduced from the equality of the moments of the forces with respect to B :

$R\ .L = F.\ \overline{PB} \Rightarrow R =\ F\ .\overline{PB}\ / L$

where $P_B = 8/\ \sin 25\ ° - y_p=1.94$ m

So : $R= 841.7\ 10^3\ 1.94 / 4 = 408.22$ kN

3- The force R required to open the door

The force R required to open the door if the weight of the door is w = 200kN.

The force R acting on A to open the door can be deduced from the equality of the moments of the forces with respect to B : $R\ L\ =\ F\ \overline{PB}$ +W 0.5L cos 25° $\Rightarrow$ $R = (\ F\ \overline{PB}$ +W 0.5L cos 25°)/ L

$R = (841.7\ 10^3\ 1.94 + 200\ 10^3\ 0.54 \cos 25\ °)/4 = 498.85$ kN.

Solution to Exercise 12

1- The Rh component

The horizontal component (Rh) of the hydrostatic force acting on the dam is equal to the sum of force F1 and the horizontal component of force F_2 (F_{2h}).

$$R_h = F_1 + F_{2h}$$

$$F_1 = \rho g h_G S = 10^3 9.81\ 5\ 10x1 = 490.5kN/m$$

$$F_{2h} = \rho g h_G Sz = 10^3 9.81\ 10.5\ 1x1 = 103.005kN/m$$

$$R_h = F_1 + F_{2h} = 593.505kN/m$$

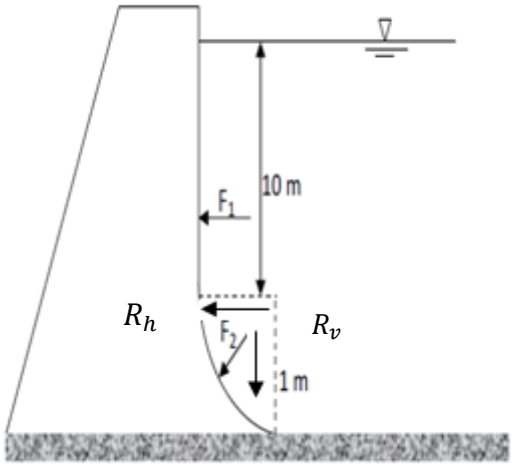
2- The Rv component

The vertical component (Rv) of the hydrostatic force exerted on the dam is equal to the weight of the liquid above the curve.:

$$R_v = F_{2V} = \rho\ g\ W = 10^3\ 9.81 \left(10x\ 1 + \frac{\pi 1^2}{4} \right) x1 = 105.8kN/m$$

3- The magnitude of the resulting force and the angle that this force makes with the horizontal.$R = \sqrt{R_h^2 + R_v^2} = \sqrt{(593.505)^2 + (105.8)^2} = 602.86kN/m$

$$tan\alpha = \frac{R_V}{R_h} = \frac{593.505}{105.8} = 0.178 \Rightarrow \alpha = 10.1^o$$



Solutions of the exercises of Chapiter II

Solution of the exercise 1

1- The volume of the displacedalcohol = The volumeof the immersed body

The immersed volume of the solid (Vi) = ¾ of the total volume of the solid (V).

V= L.l.h = 20 $x10^{-2}$x10 $x10^{-2}$x4 $x10^{-2}$= 800.10^{-6} m^3.

Vi = ¾ V = 600. 10^{-6} m^3.

The volume of the displacedalcohol= 600. 10^{-6} m^3.

1- The mass of the solid (m)

m = ρ_s .V = 600x 800x10^{-6} = 0.48 Kg

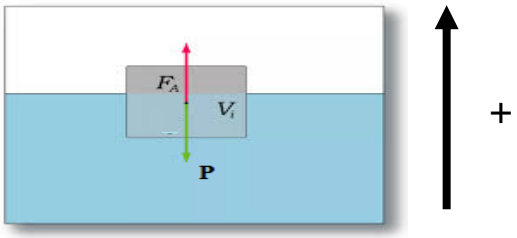
2- The solid is in equilibrium: $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_A + \vec{P} = \vec{0}$

By projection on the axis (+)

$F_A - P = 0 \Rightarrow F_A = P. \Rightarrow \rho_{Al}$ .g.Vi = m.g$\Rightarrow \rho_{Al}$ = m / Vi .

A.N.

ρ_{Al} = 0.48 / 600.10^{-6}= 800 Kg/ m^3.



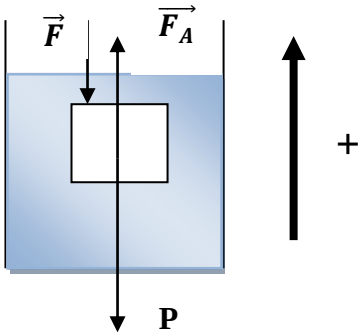
3- $F'_A = \rho_{Al}$ .g.V = 800x 800x10^{-6} x10 = 6,4 N.

4- The solid is in equilibrium:

$\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_A + \vec{P} + \vec{F} = \vec{0}$

By projection on the axis (+) :

$F_A - P - F = 0 \Rightarrow F = F_A - P$ = 6,4-4,8 =1,6 N.



Solution of the exercise 2

1- The solid is in equilibrium:

$\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{A_{eau}} + \vec{F}_{A_{Hy}} + \vec{P} = \vec{0}$

$F_{Aw} - P + F_{AHy} = 0$ **(1)**

$F_{Aw} = \rho_w . g\ V_{ie} = \rho_w . g\ a^3/5$

$F_{AHy} = \rho_{hy} . g\ V_{ie} = \rho_{hy} . g\ . 4a^3/5$

$P = \rho_s . g\ V = \rho . g\ a^3$

By replacing the forces with their expressions in (1) :

$\rho_w . g\ a^3/5 - \rho . g\ a^3 + \rho_{hy} . g\ . 4a^3/5 = 0$

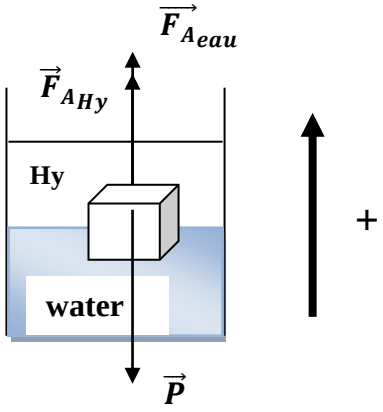
$\rho_w . /5 - \rho s + \rho_{hy} . 4/5 = 0$

$\rho_w . /5 + 4\rho_{hy} . /5 = \rho s$

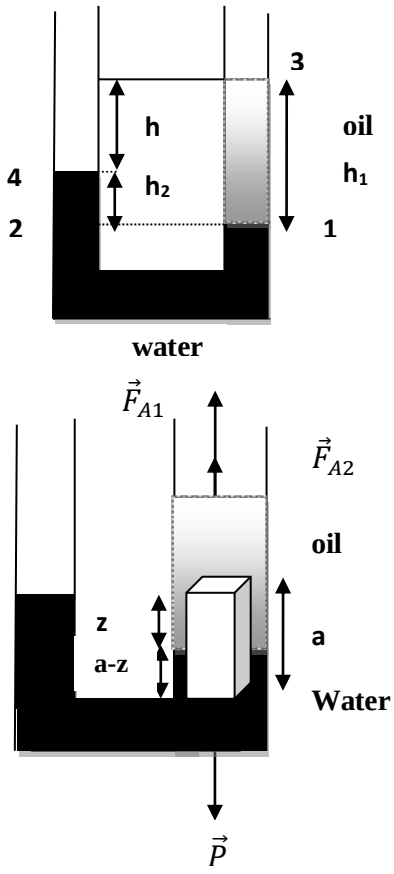
$\rho_{hy} = d\ .\ \rho_w$

$\rho_w . /5 + 4\ d\ .\rho_w /5 = \rho s$

A.N : $\rho s = 800\ Kg/m^3$



Solution of the exercise 3



$P_1 = P_2$ **(1)**
$P_1 - P_3 = \rho_h . g . h_1$
$P_1 = \rho_h . g . h_1 + P_3$
$P_3 = Patm$
$P_1 = \rho_h . g . h_1 + P_{atm}$ **(2)**
$P_2 - P_4 = \rho_{eau} . g . h_2$
$P_2 = \rho_{eau} . g . h_2 + P_4$
$P_4 = Patm$
$P_2 = \rho_{eau} . g . h_2 + P_{atm}$ **(3)**
According to the equations **(1)** , **(2)** et **(3),** we obtain :
$\rho_h . g . h_1 + P_{atm} = \rho_{eau} . g . h_2 + P_{atm}$
$h_2 = \rho_h . h_1 / \rho_{eau} .$
$\rho_h = 850$ Kg / cm^3
$\rho_{eau} = 1000$ Kg / cm^3
$h_2 = 4.25$ cm.
$h = 5 - 4.25 = 0.75$ cm.
$\rho_s = 950$ Kg / cm^3.
$\rho_s > \rho_h$, it floats at the interface of the oil
$\rho_s < \rho_{eau}$, the remaining part flows into the water.
So the body floats at the interface of separation of the two liquids
$\sum \vec{F} = \vec{0} = \vec{P} + \vec{F}_{A1} + \vec{F}_{A2} = \vec{0}$
Byprojection on the action
$-P + F_{A1} + F_{A2} = 0 \Rightarrow - m.g + \rho_w.g.S.(a-z) + \rho_h.g.S.z = 0$
$- \rho_s.g.S.a + \rho_w g.S.(a-z) + \rho_h.g.S.z = 0 \Rightarrow - \rho_s.a + \rho_w.(a-z) + \rho_{oil}.z = 0 \Rightarrow - \rho_s.a + \rho_w.a - \rho_w z + \rho_{oil}.z = 0 \Rightarrow$
$(\rho_w - \rho_s).a + (\rho_h - \rho_w) z = 0$
$z = (\rho_w - \rho_s).a / (\rho_w - \rho_h).$
$z = (1000 - 950).5.10^{-3}/(1000 - 850)$
$z = 250.10^{-3} / 150 = 1.67 . 10^{-3}$mm.

Solution of the exercise 4

$T = K . \Delta l = 50.1.9 .10^{-2} = 95 .10^{-2}$ N.
$\sum \vec{F} = \vec{0}$
$\vec{F}_{Aw} + \vec{T} + \vec{P} = \vec{0}$
Byprojection on the action+, we obtain :
$F_{Aw} + T - P = 0$
$F_{Aw} = P - T$
$F_{Aw} = 1{,}95 - 0.95 = 1N$

$F_{Aw} = \rho_w . g . Vi$
$Vi = F_{Aw} / \rho_w . g$

$$Vi = \frac{1}{10^4} = 10^{-4}m^{-3}$$

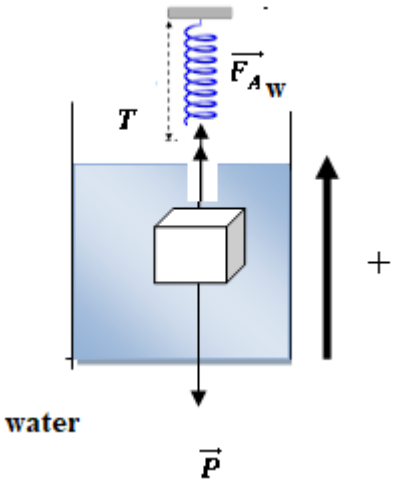
Vi $=\frac{1}{3}V_T$.

$V_T = 3.10^{-4}m^{-3}$

P = m. g = ρ_b .V_T. g $\Rightarrow \rho_b$ = P / V_T .g =1.95/ $3.10^{-3} \Rightarrow \rho_b$ = 650 Kg/m^3.

1- The spring is cut $\Rightarrow$ T = 0.

3-a $\rho_b < \rho_w$, the ball floats.



3-b

$\overrightarrow{F'_{Aw}} + \vec{P} = \vec{0}$

By projection on the axis +, we obtain :

$F'_{Aw} - P = 0$

$F'_{Aw} = P$

$F'_{Aw} = 1{,}95N$

$F'_{Aw} = \rho_w . g . Vi$

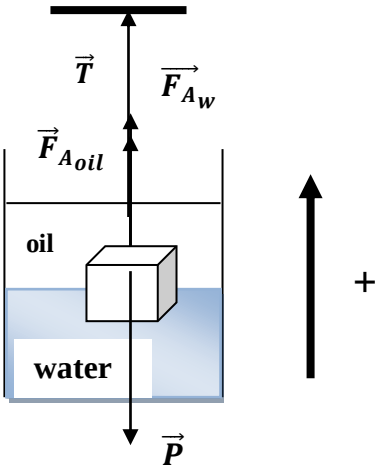
$V'i = F_{Aw}/\rho_w . g$

$$V'i = \frac{1.95}{10^{+4}} = 1.95\ 10^{-4}m^{-3}$$

Solution of the exercise 5

1- The solid is in equilibrium:

2- $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{A_{eau}} + \vec{F}_{A_{Hy}} + \vec{P} + \vec{T} = \vec{0}$



$F_{Aw} - P + F_{Aoil} + T = 0$ **(1)**

$F_{Aw} = \rho_w . g\ V_{ie} = \rho_w . g\ a^3/2$

$F_{Aoil} = \rho_{oil} . g\ V_{ie} = \rho_{oil} . g . a^3/2$

$P = \rho_s . g\ V = \rho_s . g\ a^3$

By replacing the forces with their expressions in (1) :

$\rho_w . g\ a^3/2 - \rho s . g\ a^3 + \rho_{oil} . g . a^3/2 + T = 0$

$T = -\rho_w . g\ a^3/2 + \rho . g\ a^3 - \rho_{oil} . g . a^3/2$

s

A.N :T = (2640.10 -1000.5-800.5). $(15)^3 . 10^{-6}$

T = 58.725 N

Solution of the exercise 6

1- *The density of the wooden cylinder:*

The cylinder is in equilibrium in the fluid, and it undergoes two forces: the weight P et Archimedes'sforce F_A, *donc :* $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_A + \vec{P} = \vec{0}$

$F_A - P = 0 \Rightarrow F_A = P \Rightarrow \rho_b . V_{cyl} . g = \rho_f . V_{imm} . g \Rightarrow \rho_b . S.h.g = \rho_f . S.h_1 . g$

$\rho_b . h = \rho_f . h_1 \Rightarrow \rho_b = \rho_f . h_1 / h$

2- *The modulus of force F*

The cylinder is in equilibrium but it is subjected to three forces :

$$\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_A + \vec{P} + \vec{F} = \vec{0}$$

$F_A - P + F = 0 \Rightarrow F = P - F_A \Rightarrow F = \rho_b . V_{cyl} . g - \rho_f . V_{imm} . g \Rightarrow$

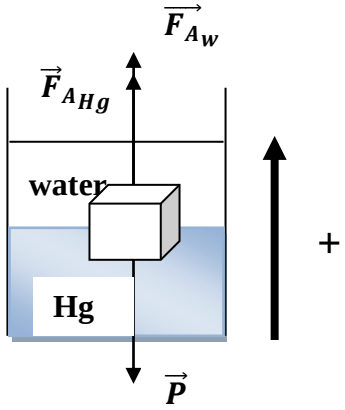
$F = \rho_b . S.h.g - \rho_f . S.(h_1 - h_2).g = \rho_f . S.h_2.g$

$F = \rho_f . \pi.R^2.h_2.g$

Solution of the exercise 7

1- *The solid is in equilibrium:*

2- $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{A_w} + \vec{F}_{A_{Hg}} + \vec{P} = \vec{0}$



$F_{Aw} - P + F_{AHg} = 0$ ***(1)***

$F_{Aw} = \rho_w . g\ V_{ie} = \rho_w . g\ S\ .(h - x_0)$

$F_{AHg} = \rho_{Hg} . g\ V_{ie} = \rho_{Hg} . g\ .S.x_0$

$P = \rho_s . g\ V = \rho s.gS.h$

By replacing the forces with their expressions in (1) :

$\rho_w . g\ S.(h - x_0) - \rho s.gS.h + \rho_{Hg} . g\ .S.x_0 = 0$

$\rho_w . (h - x_0) - \rho s.h + \rho_{Hg} . . x_0 = 0$

$\rho_w . h - \rho_w . x_0 - \rho s.h + \rho_{Hg} . . x_0 = 0$

$(\rho_w - \rho).h + (\rho_{Hg} - \rho_w).x_0 = 0$

$x_0 = (\rho_w - \rho).h / (\rho_w - \rho_{Hg})$

A.N : $x_0 = (1000 - 7100).15.10^{-2}/(1000 - 13600)$

$x_0 = 7.2610^{-2}\ m = 7.26\ cm.$

Solution of the exercise 8

Wehavetwoforces :Fa = force of Archimèdes and P : Weight of the ice block.

$\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{Aw} + \vec{P} = \vec{0}$

By projectionon the axis +, we obtain :

$F_{Aw} - P = 0 \Rightarrow F_{Aw} = P$

$F_{Aw} = \rho_w . g . V_{im}\ , P = \rho . g . V_{sphe}$

$\rho_{eau} . g . V_{im} = \rho . g . V_{sphere} \Rightarrow \frac{V_{im}}{V} = \frac{\rho_{glace}}{\rho_{eau}}.$

$A.N : \frac{V_{im}}{V} = \frac{995}{1025} = 0{,}97$

Solution of the exercise 9

At equilibrium $\Rightarrow \sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{Aeau} + \vec{P} = \vec{0}$

By projection on the axis +, we obtain :

$F_{Aw} - P = 0 \Rightarrow F_{Aw} = P$

$P = (V - 200)\rho_w g$

$P = (V - 150)\rho_{oil} g$

$$\begin{cases} P = (V - 200)x1x9{,}81 \\ P = (V - 150)x0{,}8x9{,}81 \end{cases}$$

$$\begin{cases} P = 9{,}81V - 1962 \ \ (1) \\ P = 7{,}848V - 1177{,}29 \ (2) \end{cases}$$

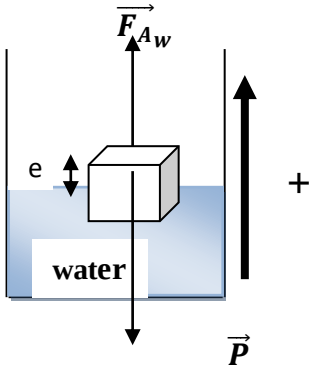
(1) = (2) $\Rightarrow$1,962 V = 784,8 $\Rightarrow$ V = 400 cm^3 et P = 1,962 N

Solution of the exercise 10

1- ρ_w = 1000 Kg / m^3 et ρ_1 = 800 Kg / m^3.

$\rho_w > \rho_1 \Rightarrow$the body floats.

2- $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{A_w} + \vec{P} = \vec{0}$



$F_{Aw} - P = 0$

$F_{Aw} = \rho w.g\ S.(h_1 - e)$

$P = \rho_1.g\ S.h_1$

$\rho_w.g\ S.(h_1 - e) - \rho_1.g\ S.h_1 = 0 \Rightarrow \rho_w.g\ S.h_1 - \rho_w.g\ S\ e - \rho_1.g\ S.h_1 = 0$

$(\rho_w - \rho_1)g\ S.h_1 - \rho_w.g\ S\ e = 0 \Rightarrow (\rho_w - \rho_1).g\ S.h_1 = \rho w.g\ S\ e$

$(\rho_w - \rho_1).h_1 = \rho_w.\ e \Rightarrow e = (\rho_w - \rho_1).\ h_1/\ \rho_w. \Rightarrow\Rightarrow\ e = (1 - \rho_1/\ \rho w).\ h_1$

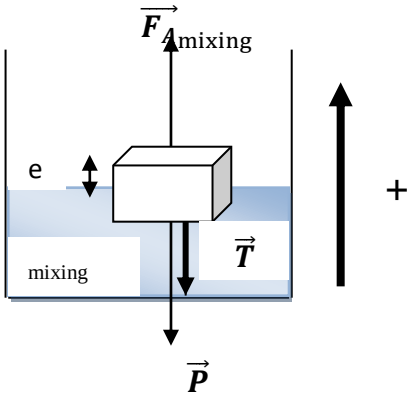
II-The density of the mixture is defined by $\rho_{mixing} = m/V$, avec $m = m_1 + m_2$ et $V = V_1 + V_2$, soit

$m = \rho_w.V_1 + \rho_{al}.V_2 = 1000.4.5\ .10^{-3} + 800.10{,}8\ .10^{-3} = 28.460$ Kg

$V = 4.5\ .10^{-3} + 10{,}8\ .10^{-3} = 30.8\ .\ 10^{-3}\ m^3$

$\rho_{mixing} = 28.460 / 30.8\ .\ 10^{-3} = 929.87$ Kg / m^3

$\rho_{mixing} > \rho_1 \Rightarrow$the body floats.



$F_{A\,mixing} - P = 0$

$F_{Aw} = \rho w.g\ S.(h_1 - e')$

$P = \rho_1.g\ S.h_1$

$\rho_{mixing}.g\ S.(h_1 - e') - \rho_1.g\ S.h_1 = 0 \Rightarrow \rho_{mixing}.g\ S.h_1 - \rho_{mixing}.g\ S\ e' - \rho_1.g\ S.h_1 = 0$

$(\rho_{mixing} - \rho_1)g\ S.h_1 - \rho_{mixing}.g\ S\ e' = 0 \Rightarrow (\rho_{mixing} - \rho_1).g\ S.h_1 = \rho_{mixing}.g\ S\ e'$

$(\rho_{mixing} - \rho_1).h_1 = \rho_{mixing}.\ e' \Rightarrow\ e' = (\rho_{mixing} - \rho_1).\ h_1/\ \rho_{mixing}. \Rightarrow e' = (1 - \rho_1/\ \rho_{mixing}).\ h_1$

3- *To further immerse the solid in water : - Connect the solid to a wire fixed to the bottom. - Exert a force to make the solid darken downwards .* $\sum \vec{F} = \vec{0} \Rightarrow \vec{F}_{A_{mixing}} + \vec{P} + \vec{T} = \vec{0}$

$F_{Amixing} - P - T = 0$

$F_{Amixing} = 9/10\ \rho.g\ S.h_1$

$P = \rho_1.g\ S.h_1$

$9/10\ \rho.g\ S.\ h_1 - \rho_1.g\ S.h_1 - T = 0 \Rightarrow 9/10.\rho.g\ S.h_1 - \rho_1.g\ S.h_1 = T$

T = (9/10.ρ.- ρ1). g S.h1
T = ((9/10). 929.87.- 800). 10 .100.10.10^{-6}
T = (836,83-800) 10^{-2} = 0.37N
If it is totally immersed:
T = (ρ.- ρ1). g S.h1 = (929.87-800). 10 .100.10.10^{-6} = (929.87-800). 10^{-2}
T= 1.29N

Solution of the exercise 11

The hydrometeris subjectedtoits ownweightof intensitym.gandto the Archimedesthrustdirectedupwardsand of intensity

ρ Vgdisplacementliquid = ρ. h.g. π. d^2/4

The equilibrium equation is: m.g = ρ. h.g. π. d^2/4

equivalent to m = ρ. h. π. d^2/4

Similarly if the liquid was water we have : m0= ρ0. h0. π. d^2/4

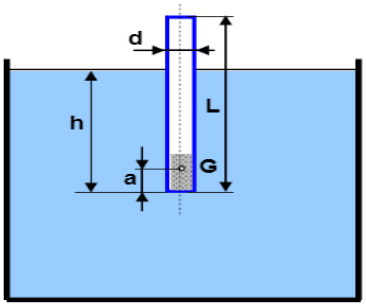
(1)et (2) drives ρ0.h0 = ρ . h

$\rho = \rho_0.h_0/ h = 800$ Kg / m^3

1- *The density of mass ρ min*

correspondstoa submergedheight h = 400 mm

$\rho_{min} = \rho_0.h_0/ h = 500$ Kg / m^3



The hydrometer remains in a stable equilibrium position if the center of gravity of the liquid is displaced (locatedat a distance h/2 from the base) is above the center of gravity (located at a distance a from the base).

So, it is necessary that h / 2 > a
to ensure the stability of the density meter ρ<ρ0.h0/ 2a
ρ<10000 Kg/m^3

Mercury has a density ρ = 13600 Kg/m^3> 10000 Kg/m^3

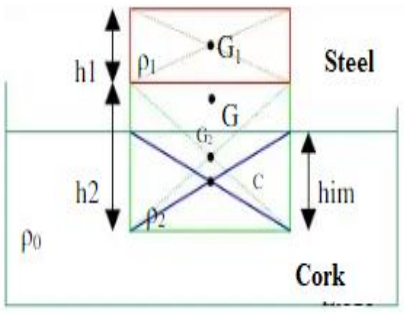
Solution of the exercise 12

For the solid to float, it is necessary
$\rho_s < \rho_L$
$\rho_L = 10^3$ Kg /m^3

$$\rho_s = \frac{M_s}{V_s} = \frac{M_1 + M_2}{V_1 + V_2}$$

M_1 : mass of steel; M_2 : cork mass.
V_1 : volume de steel; V_2 : volume of cork.

$$\rho_s = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$



$$\rho_s = \frac{\rho_1 h_1 S + \rho_2 h_2 . S}{h_1 S + h_2 S} = \frac{\rho_1 h_1 + \rho_2 h_2}{h_1 + h_2} = \frac{(7800x2) + (150x20)}{2+20} = 845.5\, Kg/m^3$$

$\rho_s = 845.5\, Kg/m^3 < \rho_L \Rightarrow$ The solid will float

To show stability or instability, it is necessary to determine the positions of G and C

OG = ?

$OG_2 = h_2/2$

$OG_1 = h_2 + h_1/2$

$$OG = \frac{M_1 OG_1 + M_2 OG_2}{M_1 + M_2} = \frac{S.\rho_1 h_1 . OG_1 + S.\rho_2 h_2 . OG_2}{S.\rho_1 h_1 + S.\rho_2 h_2}$$

$$\frac{\rho_1 h_1 . (h_2 + \frac{h_1}{2}) + \rho_2 h_2 . \frac{h}{2}}{\rho_1 h_1 + . \rho_2 h_2}$$

$$OG = \frac{7800x.(20+2) + (150x20x10)}{(7800x2) + (150x20)} = 19.2\, cm$$

C Geometric center of the submerged part (Vim = ?)

$$\sum \vec{F} = \vec{0}$$

$$\vec{F}_{Aw} + \vec{P} = \vec{0}$$

$P = F_{A\,w}$

$P = \rho_s V_s . g$

$F_{A\,w} = \rho_L V_{im} . g$

$\rho_s V_s = \rho_L V_{im}$.

$V_{im} = \rho_s V_s / \rho_L$

$$V_{im} = \frac{\rho_s (V_1 + V_2)}{\rho_0} = \frac{\rho_s (S.h_1 + S.h_2)}{\rho_0}$$

$$V_{im} = \frac{845.5(20x2 + 20x20)}{1000} = 372\ cm^3$$

$V_{im} = S h_{im}$

$$h_{im} = \frac{V_{im}}{S} = 18.6\ cm$$

where $OC = \frac{h_{im}}{2} = 9.3\ cm$

wehave OG = 19.2 cm

Conclusion : G is above C, the balance can be stable or unstable.

The position of M (Metacenter) is determined

$CM = I/V_{im}$

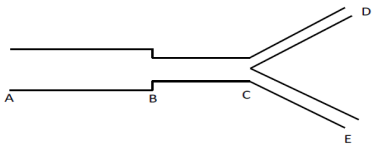
Waterline Surface is a circle $I = \pi . D^4/64$

$I = \pi . 16S^2/\pi^2/64 = S^2/4\pi$

CM = $S^2/4\pi/S.h_{im}$ =S/ 4π.him = 20 /4π.18.6
= 0.0855 cm
CG = OG –OC = 19.2 -9.3 = 9.9 cm
Conclusion CM < CG →Unstable position

Solutions of the exercises of chapiter 3

Solution of the exercise 1



1-$Q = V.S = V.\frac{\pi d_1^2}{4} = 2\frac{3.14x(1.5)^2}{4} = 3.53m^3/s$

2- $Q = V_{BC}.S_{BC} \Rightarrow V_{BC} = \frac{4Q}{\pi d_2^2} = \frac{4x3.53}{3.14x1.2^2} =$

$3.12m/s$

3-$Q_{CD} = \frac{Q}{3} = 1.176m^3/s \Rightarrow V_{CD} = \frac{4Q_{CD}}{\pi d_3^2} =$

$\frac{4x1.176}{3.14x0.8^2} = 2.34m/s$

4- $Q = Q_{CD} + Q_{CE} = \frac{2}{3}Q = 2.354m^3/s$

$$Q_{CD} = V_{CE}.S_{CE} = V_{CE}.\frac{\pi d_4^2}{4} \Rightarrow d_4 = \sqrt{\frac{4Q_{CE}}{\pi.V_{CE}}}$$

$$= 1.095m$$

Solution of the exercise 2

We apply the continuity equation

$V_1.S_1 = V_2.S_2$ ou encore $S_1/S_2 = V_2/V_1$

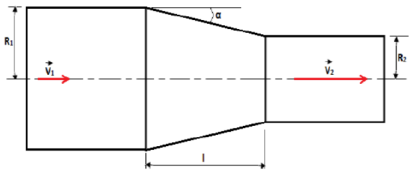
$S_1 = \pi R_1^2$, $S_2 = \pi R_2^2$

where $\frac{R_1}{R_2} = \sqrt{\frac{V_2}{V_1}} = 2$

2-$tg\alpha = \frac{R_1 - R_2}{l}$ donc $l = \frac{R_1 - R_2}{tg\alpha}$

or $R_2 = \frac{R_1}{2}$ donc $l = \frac{R_1}{2\,tg\alpha}$.

A.N. l = 93.3 mm



Solution of the exercise 3

Flow speed: $V = \frac{q_v}{S} = \frac{4.q_v}{\pi.d^2}$ A.N $V = \frac{4.0.4.10^{-3}}{\pi.0.01^2} = 5.1\ m/s$

Bernoulli's theorem: $P_1 + \rho\frac{V_1^2}{2} + \rho.gZ_1 = P_2 + \rho\frac{V_2^2}{2} + \rho.gZ_2$

We have z_1-z_2 = h ; $P_1 = P_2 = P_{atm}$; $V_1 = 0$ $h = \frac{V_2^2}{2.g}$ A.N $h = \frac{5.1^2}{2.9.81} = 1.32m$

Solution de l'exercice 4

Let's apply Bernoulli's equation between 1 and 2 $P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g z_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g z_2$

$$P_1 - P2 = \frac{1}{2}\rho v^2{}_2 - \frac{1}{2}\rho v^2{}_1 + \rho g(z_2 - z_1)$$

$P_1 = P_2$

$$\frac{1}{2}\rho v^2{}_2 - \frac{1}{2}\rho v^2{}_1 + \rho g(z_2 - z_1) = 0$$

We apply the continuity equation

$V_1.S_1 = V_2.S_2$ or again $V_2 = V_1.S_1/S_2$

$S_1 = \pi d_1^2/4$, $S_2 = \pi d_2^2/4$

$V_2 = (D_1{}^2/D_2{}^2).V_1 = \left(\frac{D_1}{D_2}\right)^2 . V_1$

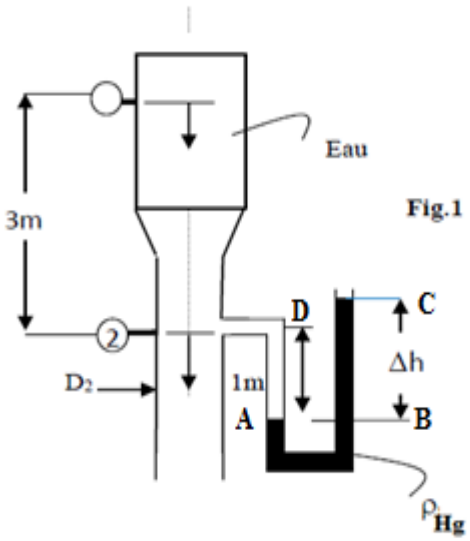
$$\frac{1}{2}\rho v^2{}_1\left(1 - \left(\frac{d_1}{d_2}\right)^4\right) = -\rho g(z_2 - z_1) \Rightarrow$$

$$\frac{1}{2}\rho v^2{}_1\left(\left(\frac{d_1}{d_2}\right)^4 - 1\right) = \rho g(z_2 - z_1)$$

$$\left(\frac{d_1}{d_2}\right)^4 = \frac{2.g.(z_2 - z_1)}{v_1^2} + 1$$

$$\left(\frac{d_2}{d_1}\right)^4 = \frac{v_1^2}{2.g.(z_2 - z_1) + v_1^2}$$

$$d_2 = d_1.\sqrt[4]{\frac{v_1^2}{2.g.(z_2 - z_1) + v_1^2}}$$



The pressures P_1 and P_2 are measured using manometer containing density mercury $\rho_{Hg.}$ it is considered that the pressure gauge is in hydrostatic equilibrium. So there is no flow.Sowe can calculate the pressuresusingthe laws of hydrostatics.

$P_A = P_B$ **(1)**

$P_A - P_D = \rho_e.g.h_1$ **(2)**, $h_1 = 1$ m

$P_A = \rho_e.g.h_1 + P_D$, $P_D = P_2$

$P_B - P_C = \rho_h.g.\Delta h$

$P_B = \rho_h.g.\Delta h + P_C$ **(3)** , $P_C = P_{atm}$

According to the 3 equations, we obtain

:$\rho_e.g.h_1 + P_2 = \rho_h.g.\Delta h + P_{atm}$

$$\Delta h = \frac{\rho_e.g.h_1 + P_2 - P_{atm}}{\rho_h.g} .$$

A.N: $\Delta h = \frac{10^3.10.1 + 12x10^4 - 10^5}{13600.10} = 0.22m$

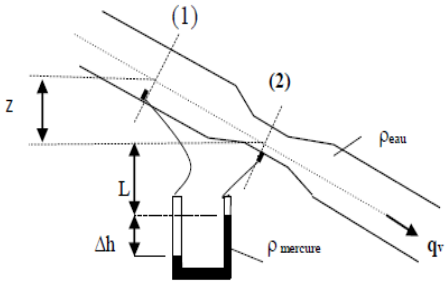
Solution of the exercise 5

Let's apply Bernoulli's equation between 1 and 2 $P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g z_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g z_2$

$$P_1 - P2 = \frac{1}{2}\rho v^2{}_2 - \frac{1}{2}\rho v^2{}_1 + \rho g(z_2 - z_1)$$

$$P_1 - P2 = \frac{1}{2}\rho v^2{}_2 - \frac{1}{2}\rho v^2{}_1 - \rho g Z \quad \textbf{(1)}$$

We have Z_1-Z_2 = Z



Inthe nanotube :

$P_A = P_B$

$$P_A - P_1 = \rho_w . g(\Delta h + L + z)$$

$$\Rightarrow \quad P_A = \rho_w . g(\Delta h + L + z) + P1$$

$$P_B - P_2 = \rho_{Hg} . g . \Delta h + \rho_w . g(L)$$

$$\Rightarrow P_B = \rho_{Hg} . g . \Delta h + \rho_w . g(L) + P_2$$

$$P_1 - P_2 = \rho_{Hg} . g . \Delta h + \rho_w . g(L) - \rho_w . g(\Delta h + L + z) = \rho_{Hg} . g . \Delta h - \rho_w . g . \Delta h - \rho_w . g . Z \quad \textbf{(2)}.$$

From (1) et (2) we have :

$$\frac{1}{2}\rho V^2{}_2 - \frac{1}{2}\rho V^2{}_1 - \rho g Z = \rho_{Hg} . g . \Delta h - \rho_w . g . \Delta h - \rho_w . g . Z$$

$$\frac{1}{2}\rho_w V^2{}_2 - \frac{1}{2}\rho_w V^2{}_1 = \left(\rho_{Hg} - \rho_w\right) . g . \Delta h$$

$$V_2 = \frac{S_1}{S_2} . V_1$$

$$\frac{1}{2}\rho_w \left(\left(\frac{S_1}{S_2}\right)^2 - 1\right) . V_1{}^2 = \left(\rho_{Hg} - \rho_w\right) . g . \Delta h$$

$$\Delta h = \frac{\frac{1}{2}\rho_w \left(\left(\frac{S_1}{S_2}\right)^2 - 1\right) . V_1{}^2}{\left(\rho_{Hg} - \rho_w\right) . g}$$

$$\frac{1}{2}\rho_{eau}\left(\left(\frac{S_1}{S_2}\right)^2-1\right).V_1{}^2=\left(\rho_{Hg}-\rho_w\right).g.\Delta h$$

$$S_1=\pi.D_1{}^2\ ,S_2=\pi.D_2{}^2$$

$$\frac{1}{2}\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right).V_1{}^2=\left(\rho_{Hg}-\rho_w\right).g.\Delta h$$

$Q_v = S_1.V_1 \Rightarrow V_1=Q_v / S_1 = Q_v /\pi.D_1{}^2$

$$\frac{1}{2}\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right).\left(\frac{Qv}{\pi.D_1{}^2}\right)^2$$

$$=\left(\rho_{Hg}-\rho_w\right).g.\Delta h$$

$$\frac{1}{2}\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right).\frac{Qv^2}{\pi^2.D_1{}^4}$$

$$=\left(\rho_{Hg}-\rho_w\right).g.\Delta h$$

$$\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right).Qv^2$$

$$=2.\pi^2.D_1{}^4\left(\rho_{Hg}-\rho_w\right).g.\Delta h$$

$$Qv^2=\frac{2.\pi^2.D_1{}^4\left(\rho_{Hg}-\rho_w\right).g.\Delta h}{\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right).}$$

$$Q_v=\sqrt{\frac{2.\pi^2.D_1{}^4\left(\rho_{Hg}-\rho_w\right).g.\Delta h}{\rho_w\left(\left(\frac{D_1}{D_2}\right)^4-1\right)}}$$

A.N : Δh = 90mm et D_1= 200mm et D_2= 90 mm

Solution of the exercice 6

1-By applying Bernoulli's theorem between A and s:

$$\frac{\rho v_s^2}{2}+P_s+\rho.g.z_s=\frac{\rho v_A^2}{2}+P_A+\rho.g.z_A$$

$P_s = P_A = P_{atm}$

$v_A = 0$

$Z_A - Z_s = H$

$\boldsymbol{v_s} = \sqrt{2gH}$

A.N :

$v_s = \sqrt{2x9.81x3} = 7.67 m/s$

1- Volume flow rate $Q_v = v_s . \frac{\pi.d^2}{4}$

A.N : $Q_v = 7.67x \frac{\pi.0.01^2}{4} =$

$6.025x\ 10^{-4} m^3/s = 0.602$ l/s

3-By applying Bernoulli's theorem between M

and S: $\frac{\rho v_M^2}{2} + P_M + \rho . g . z_M = \frac{\rho v_s^2}{2} + P_s + \rho . g . z_s$

$P_s = P_{atm}$

$v_M = v_s$

$Z_M - Z_s = H + h$

$P_M = P_{atm} - \rho . g(. z_M - . z_s)$

$= P_{atm} - \rho . g(. H + \square)$

$P_M = 10^5 - 10^3 . g(. H + \square)$

$P_M = 10^5 - 10^3 . 9.81x(3 + 0.4)$

$= 74{,}494x10^3 Pa$

4- No. It is necessary that PM is greater than

0.which is equiavalent to h< $\frac{P_{atm}}{\rho.g} + H$

$h < \frac{10^5}{10^3.9.81} + 3 = 13.19$ m , $h < 13.1m.$

Solution of the l'exercise 7

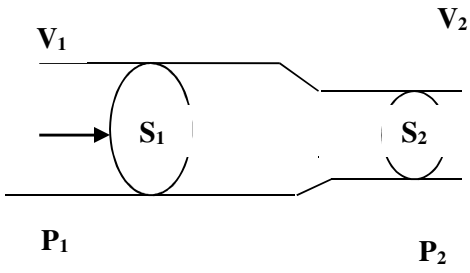
$Q_{v1} = Q_{v2}$

$S_1 . V_1 = S_2 . V_2$

$S_1 = \pi . D_1^{\ 2}$, $S_2 = \pi . D_2^{\ 2}$

$V_2 = (S_1/S_2) . V_1$

$V_2 = (D_1^{\ 2}/D_2^{\ 2}) . V_1 = \left(\frac{D_1}{D_2}\right)^2 . V_1$



A.N : $V_2 = 4$. V1=0.4m / s

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g z_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g z_2$

$z_1 = z_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 = P_2 + \frac{1}{2}\rho v^2{}_2$

$$P_2 = P_1 + \frac{1}{2}\rho {v^2}_2 - \frac{1}{2}\rho {v^2}_1$$

$$P_2 = P_1 + \frac{1}{2}\rho({v^2}_1 - {v^2}_2)$$

$$P_2 = P_1 + \frac{1}{2}\rho({v^2}_1 - {v^2}_2)$$

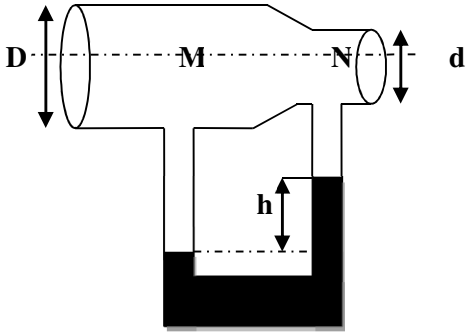
A.N :

$$P_2 = 10^3 + \frac{1}{2}10^3(0.01 - 0.16)$$

$$P_2 = 10^3 + 500.(-0.15)$$

$P_2 = 925$ Pa

Solution of the exercise 8



1-$Q_{vM} = Q_{vN}$

$S_M V_M = S_N . V_M$

$S_M = \pi . D^2$, $S_N = \pi . d^2$

$V_N = (S_M/S_N).V_M$

From **(I) and (II)**

$\left(\rho_w - \rho_{Hg}\right) g.h = \frac{1}{2}\rho_w {v^2}_M (1 - \left(\frac{D}{d}\right)^4)$

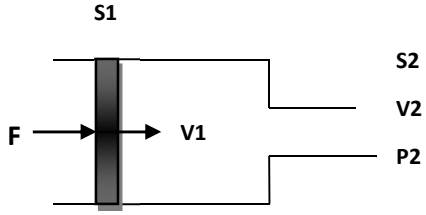
$$v^2_M = \frac{2\left(\rho_w - \rho_{Hg}\right)}{\rho_w.(1 - \left(\frac{D}{d}\right)^4)}.g.h$$

$$V_M = \sqrt{\frac{2\left(\rho_w - \rho_{Hg}\right)}{\rho_w.(1 - \left(\frac{D}{d}\right)^4)}.g.h}$$

Solution of the exercise 9

1-

$$P_1 = \frac{F}{S_1} + P_{atm}$$



$$S_1 = \pi.\left(\frac{d_1}{2}\right)^2 = 12.56x10^{-4}$$

$$P_1 = \frac{62.84}{12.56x10^{-4}} + 10^5 = 5x10^4 + 10^5$$

$$= 1.5x10^5 Pa$$

2-

$$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g z_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g z_2$$

$z_1 = z_2$

$$P_1 + \frac{1}{2}\rho v^2{}_1 = P_2 + \frac{1}{2}\rho v^2{}_2$$

$S_1 V_1 = S_2 . V_2$

$S_1 = \pi . d_1{}^2$, $S_N = \pi . d_2{}^2$

$V_1 = (S_2/S_1).V_2$

$V_1 = (d_2{}^2/d_2{}^2).V_1 = \left(\frac{d_2}{d_1}\right)^2 . V_2$

$$P_1 - P_2 = \frac{1}{2}\rho . (v^2{}_2 - v^2{}_1)$$

$$= \frac{1}{2}\rho v^2{}_2 (1 - \left(\frac{D}{d}\right)^4)$$

$$v^2{}_2 = \frac{2(P_1 - P_2)}{\rho . (1 - \left(\frac{d_2}{d_1}\right)^4)}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho . (1 - \left(\frac{d_2}{d_1}\right)^4)}}$$

A.N :

$$V_2 = \sqrt{\frac{2(1.5 - 1)10^5}{10^3 . (1 - \left(\frac{0.5}{2}\right)^4)}} = 10.17 m/s$$

3- $Q_v = S_2 . V_2$, $S_2 = \pi . \left(\frac{d_2}{2}\right)^2 = 0.19625 \text{x} 10^{-4}\ m^2$

$Qv = 1.995\ \text{x}10^{-4} m^3/s$

Solution of the exercise 10

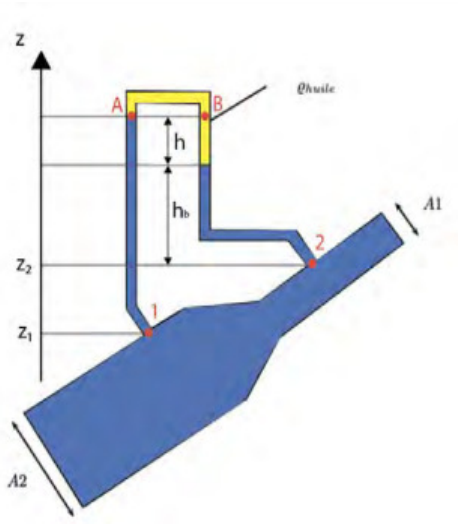
Let's apply Bernoulli's theorem at two points, 1 and 2, located along a current line :

$$P_1 + \frac{1}{2}\rho_e v^2{}_1 + \rho_e g z_1$$

$$= P_2 + \frac{1}{2}\rho_e v^2{}_2 + \rho_e g z_2$$

$$P_1 - P_2 = \frac{1}{2}\rho_e (v^2{}_2 - v^2{}_1) + \rho_e g (z_2 - z_1) \quad \textbf{(1)}$$

The pressures P1 and P2 are measured using a manometer containing oil with a density of mass $\rho_h<\rho_w$. it is considered that the pressure gauge is in hydrostatic equilibrium. Indeed, the pipes are thin. So there is no flow. Il n'yadonc pas d'écoulement. So there is no flow. So we can calculate P_1 et P_2using the laws of hydrostatics.$P_A= P_B$



$P_1-P_A = \rho_e.g.(z_1-z_2)+ \rho_e.g.h_b+\rho_e.g.h$

$P_2-P_B = \rho_e.g.h_b+\rho_h.g.h$

$P1-P2=\rho_e.g.(z_1-z_2)+ \rho_e.g.h_b-\rho_h.g.h$ (2)

By combining equation (2) and equation (1),

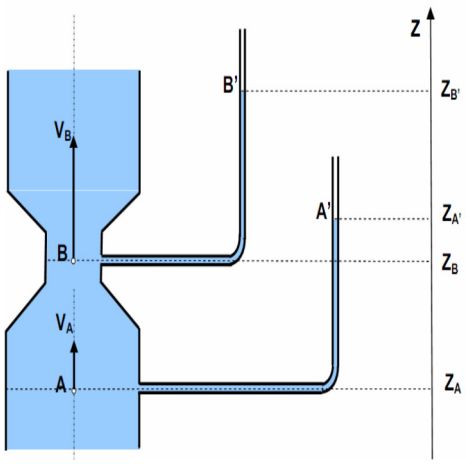
we have :$\frac{1}{2}\rho_e.(v^2{}_2 - v^2{}_1) + \rho_h.g.(z_2 - z_1) ==$

$\rho_e.g.(z1 - z2) + \rho_e.g.hb - \rho_h.g.h$

$\frac{1}{2}\rho_e.(v^2{}_2 - v^2{}_1) = (\rho_e - \rho_h).g.h$

$$h = \frac{\rho_e.(v^2{}_2 - v^2{}_1)}{2(\rho_e - \rho_h).g}$$

Correction of the exercise 11



1- *Fundamental Relationship of Hydrostatics) between B and B' :*

$P_B-P_{B'} = \rho.g\ (Z_{B'}-Z_B) \Rightarrow P_B = \rho.g.(Z_{B'}-Z_B) + P_{B'}$

A.N : $P_B = 1000.9{,}8.(2{,}541-0{,}5) +10^5 = 1{,}2$ *bar*

2*- Fundamental Relationship of Hydrostatics) between A and A' :*$P_A - P_{A'} = \rho.g\ (Z_{A'}-Z_A) \Rightarrow P_A = \rho.g.(Z_{A'}-Z_A) + P_{A'}$

A.N : $P_A = 1000.9{,}81.(3{,}061-0) +10^5 =$ *1,3 bar*

3*-Continuityequation:*$S_A.V_A = S_B.V_B \Rightarrow V_B = (S_A/S_B).V_A = V_A.(d_A/d_B)^2 = 4.V_A$

$4\text{-}P_A + \frac{1}{2}\rho_e v^2{}_{1A} + \rho_e g z_A = P_B + \frac{1}{2}\rho_e v^2{}_B +$

$\rho_e g z_B$

$$P_A - P_B + \rho_e . g . (z_A - z_B) = \frac{1}{2} . \rho_e . (15/16) . v^2{}_B$$

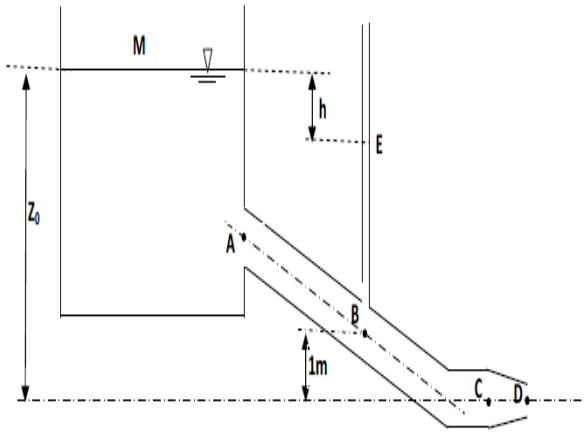
$$v_B = \sqrt{\frac{16}{15} . 2 . \left(\frac{P_A - P_B}{\rho_e} + g . (z_A - z_B)\right)}$$

$$= \sqrt{\frac{16}{15} . 2 . \left(\frac{1{,}3 - 1{,}4}{1000} + 9{,}81(0 - 0{,}5)\right)}$$

Correction of the exercise 12

Output speed V_D

Let's apply Bernoulli's theorem at two points, M and D. $P_M + \frac{1}{2}\rho_e v^2{}_M +$ $\rho_e . g . z_M = P_D + \frac{1}{2}\rho_e v^2{}_D + \rho_e . g . z_D$



$P_M = P_D$=Patm , $V_M = 0$ (large tanksize), $Z_M = Z_0$= 4m , Z_D= 0

$$V_D = \sqrt{2 . g . Z_M} = \sqrt{2.9{,}81.4} = 8{,}859\ m/s$$

1- DebitQv

$Q_V = S_D . V_D = \pi . \frac{0{,}02^2}{4} 8{,}859 = 0{,}0028\ m^3/s$

2- SpeedV$_c$

$Q_V = S_D . V_D = S_c . V_c \Rightarrow V_C = V_D . (d_D / d_C)^2 =$ 8,859. $(0{,}02/0{,}05)^2$ = 1,417m/s

3- Presion P_B

Let's apply Bernoulli's theorem at two points, B and M : $P_M + \frac{1}{2}\rho_e v^2{}_M +$ $\rho_e . g . z_M = P_B + \frac{1}{2}\rho_e v^2{}_B + \rho_e . g . z_B$

$Z_M = Z_0$= 4m , Z_B= 1.0 m, $V_B = V_C$ = 1,417 m/s et V_M = 0 m/s, P_M =Patm

$$P_B = P_M + \frac{1}{2}\rho_e v^2{}_B + \rho_e . g . (z_M - z_B)$$

$$P_B = 10^5 + \frac{1}{2}10^3.1{,}417^2 + 10^5.9{,}81(4-1) = 1034{,}3745\ Pa$$

The height By application of the law of hydrostaticsinthe piezometer

P_B-$Patm$ = ρ.g.(Z_0-Z_B-h)

h = P_B-$Patm$- ρ.g.(Z_0-Z_B) = $\frac{1}{2}\frac{V_B^2}{g}$ =

$0{,}5\ x\frac{1{,}417^{\ 2}}{9{,}81} = 0{,}102\ m$

Solution of exercises of chapiter IV

Solution of the exercise 1

To determine the flow regime, the Reynolds number is calculated

$$R_e = \frac{V.D}{\nu} = \frac{\rho.V.D}{\eta}$$

1- **A.N :** $R_e = \frac{10.5\ x3x10^{-2}}{10^{-6}} = 31.5x10^4 > 4000$ ⇒ the regime is turbulent

2- A.N : $R_e = \frac{10.5\ x3x10^{-2}}{110x10^{-6}} = 2860 > 2000$ ⇒ the diet is critical

3- A.N : $R_e = \frac{10.5\ x3x10^{-2}}{290x10^{-6}} = 1086 < 2000$ ⇒ the regime is laminar

Solution of the exercise 2

1- **The kinematic viscosity ν**

$\nu = \frac{\eta}{\rho_f}$, ρ_f = d. ρeau = 0.932 x 1000=932 Kg/m^3 , $\nu = \frac{0.11}{932} = 1.18\ x10^{-4}\ m^2/s$

2- **The flow speed V**.

qv = S.V , V = qv /S , S = π . D^2/4 =3.14 12.5^2 10^{-4}= 490.625x10^{-4} m^2

V= 0.4 m/s

3- **The Reynolds number Re.**

$R_e = \frac{\rho.V.D}{\eta} = \frac{932.0.4.25x10^{-2}}{0.11} = 847{,}27 < 2000$

The nature of the flow.

$R_e < 2000$, the flow is laminar.

The pressure drop in the pipe

$\Delta H = \lambda \frac{\rho V^2}{2}.\frac{L}{D}$, $\lambda = \frac{64}{R_e}$,**A.N :** $\lambda = 0.075$

$$\Delta H = 0.075 \frac{932(0.4)^2}{2}.\frac{1650}{25x10^{-2}} = 7{,}38144\ 10^4 Pa$$

Solution of the exercise 3

1- The pressure difference P_1-P_2

Let's apply Bernoulli's equation between 1 and 2 $P_1 + \frac{1}{2}\rho_h v^2{}_1 + \rho_h g z_1 = P_2 + \frac{1}{2}\rho_h v^2{}_2 + \rho_h g z_2 + \Delta H_{12}$

$v_1=v_2$, $z_1=z_2$

$P_1 - P_2 = \Delta H_{12}$ **(I)**

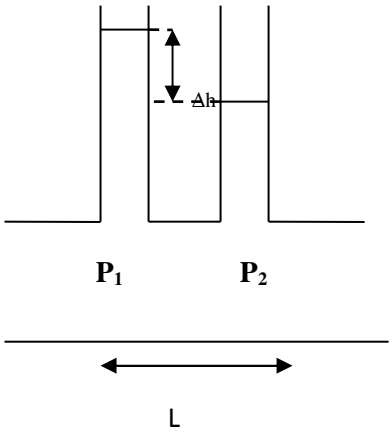
In the nano tube :

$P_1 = P_2$ **(1)**

$P_1 - P_A = \rho_h . h_1 \Rightarrow P_1 = \rho_h . g . h_1 + P_A$ **(2)**

$P_2 - P_B = \rho_h . g . h_2 \Rightarrow P_2 = \rho_h . g . h_2 + P_B$ **(3)**

$P_1 - P_2 = \rho_h . g . \Delta h$ **(II)**



2- The dynamic viscosity

From **(I) and (II)** , $\Delta H_{12} = \rho_h . g . \Delta h$ **(III)**

In the case of laminar flow :

$\Delta H_{12} = \frac{8.\eta.L.Q_v}{\pi R^4}$ **(IV)**

According to III and IV, on a : $\frac{8.\eta.L.Q_v}{\pi R^4} = \rho_{eau} . g . \Delta h \Rightarrow \eta = \frac{\rho_h . g . \Delta h . \pi R^4}{8.L.Q_v}$

A.N : $\eta = \frac{91..267.3.143.5^4.10^{-4}}{8x600x0,4} = 0,0596$ Pe

The nature of flow

To determine the nature of the flow, the Reynolds number is calculated Re.

$$R_e = \frac{\rho . V . D}{\eta}$$

We calculate the speed V

$Q_V = S.V$

$S = \pi.R^2 = 3.14x3.5^2 \times 10^{-6} = 38.456 \times 10^{-6}$

$V = Q_v / S = 0.1$ m / s

$R_e = \frac{910x0.1x.7x10^{-3}}{0.0596} = 10.69 < 2000$, the flow is laminar.

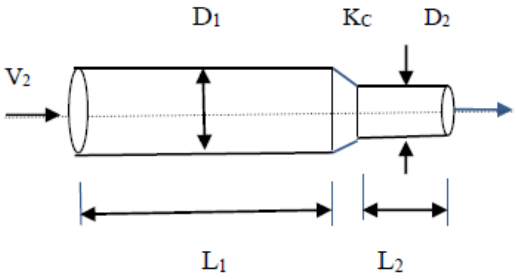
Solution of the exercise 4

The pressure difference between the inlet and the outlet of the pipe $P_1 - P_2 =$

$$\rho \frac{v_1^2}{2}\left(\left(\frac{D_1}{D_2}\right)^4 - 1\right) + \rho g(z_2 - z_1) + \rho\, g\Delta H_{12}$$

$$\Delta H_{12} = \Delta H_1 + \Delta H_2$$

$$\Delta H_1 = \lambda_1 \frac{L_1.V_1^2}{2gD_1} \quad , \quad \Delta H_2 = \lambda_{21} \frac{L_2.V_2^2}{2gD_2}$$



$$,V_2 = \frac{V_1.S_1}{S_2} = V_1\left(\frac{D_1.}{D_2}\right)^2 = 0.5x\left(\frac{20}{5}\right)^2 = 8m/s$$

$$Re_1 = \frac{V_1.D_1}{\nu} = \frac{0.5\ x\ 0.02}{10^{-5}} = 1000 \quad , Re_2 = \frac{V_2.D_2}{\nu} = \frac{8\ x\ 0.005}{10^{-5}} = 4000$$

$\mathrm{Re}_1 < 2000, so$ laminar flow , $\mathrm{Re}_2 > 2000, so$ turbulent flow

$$\lambda_1 = \frac{64}{Re_1} = \frac{64}{1000} = 0.064 \quad , \lambda_2 = 0.136.(Re_2)^{-1/4} = 0.0397$$

$$\Delta H_{12} = \lambda_1 \frac{\rho.L_1.V_1^2}{2D_1} + \lambda_2 \frac{\rho.L_2.V_2^2}{2D_2}$$
$$= 0.20 + 5.18 = 5.38m$$

AN:

$$P_1 - P_2 = \rho \frac{v_1^2}{2}\left(\left(\frac{D_1}{D_2}\right)^4 - 1\right) + \rho\, g\Delta H_{12} =$$
$$= 850\ \frac{(0.5)^2}{2}\left(\left(\frac{20}{5}\right)^4 - 1\right)$$
$$+ 850x9.81x5.38$$
$$= 71991\ Pa$$

$$\lambda \frac{\rho V^2}{2}.\frac{L}{D}$$

Solution of the exercise 5

The kinematic viscosity of the oil

$\nu = \frac{\eta}{\rho}$, $\nu = \frac{0.11}{900} = 1.22x10^{-4}m^2/s$

1- he flow velocity V and the mass flow Qm

$$V = \frac{4\ x\ 19.7x10^{-3}}{\pi(0.25)^2} = 0.40m/s$$

$Q_m = \rho \times Q_v = 0.9 \times 10^3 \times 19.71 \times 10^{-3} = 17{,}73$ Kg/s

The Reynolds number and the nature of the flow

$$R_e = \frac{V.D}{\nu} = \frac{0.4x0.25}{1.22x10^{-4}} = 820$$

$R_e < 2000$, the flow is laminar and $\lambda = \frac{64}{R_e} = \frac{64}{820} = 0.078$

The linear pressure drop coefficient and the pressure drop in the pipe.

$$\Delta H = \lambda \frac{\rho V^2}{2}.\frac{L}{D} = 0.078 \frac{900x1650x(0.4)^2}{2x0{,}25} = 4.2Pa$$

Solution of the exercise 6

1. Calculate the average speed v.

$v = Qv / S$; $S = \pi.D^2 / 4$; $S = 5{,}0265\times10^{-3}\ m^2$

$v = 0{,}398 m/s$

2.Calculation of the Reynolds number and the flow rate $R_e = \frac{V.D}{\nu}$

$R_e \approx 31840$

Puisque Re>4000Re > 4000Re>4000, the flow is turbulent.

3. Calculation of the friction factor with the de Blasius formula.

$\lambda = 0.316 . Re^{-1/4}$, $\lambda \approx 0{,}0237$

4. Calculation of the linear pressure drop ΔP.

Solution of the exercise 7

1- Pressure drop coefficient λ

Bernoulli's equation :

$$P_1 + \frac{1}{2}\rho_h v^2{}_1 + \rho_h g z_1 = P_2 + \frac{1}{2}\rho_h v^2{}_2 + \rho_h g z_2 + \Delta H_{12}$$

$$\boldsymbol{v_1 = v_2} \quad \textbf{et } \boldsymbol{z_1 = z_2}\textbf{ , } \Delta H = \lambda \frac{\rho.V^2}{2}.\frac{\boldsymbol{L}}{\boldsymbol{D}}$$

$$P_1 = P_2 + \Delta H_{12} \Rightarrow P_1 - P_2 = \lambda \frac{\rho L.V^2}{2D}$$

$$\lambda = \frac{2x\Delta P}{\rho x V^2}.\frac{D}{L} = \frac{2x1.5x10^3}{961x1.5^2}.\frac{0{,}120}{130} = 0.128$$

The Reynolds number if the flow is laminar

$$\lambda = \frac{64}{R_e} \Rightarrow R_e = \frac{64}{\lambda} = \frac{64}{0{,}128} = 500$$

2- Kinematic viscosity coefficient ν

$$R_e = \frac{V.D}{\nu} \Rightarrow \nu = \frac{V.D}{R_e}\ A.N : \nu = \frac{1.5x0.12}{500} = 36\,x10^{-5} m^2/s$$

Solution of the exercise 8

Let's apply Bernoulli's equation between 1 and 2:

$$P_1 + \frac{1}{2}\rho_{eau} v^2{}_1 + \rho_{eau} g z_1 = P_2 + \frac{1}{2}\rho_{eau} v^2{}_2 + \rho_{eau} g z_2 + \Delta H_{12}$$

$z_1 = 0, z_2 = \Delta z$, P_1=P_2 = Patm , $v_1 = v_2 = 0$ (Constant water level) , so :

$$\rho_{eau} g z_2 = \Delta H_{12}$$

$$\Delta H = \lambda_1 \frac{\rho.v_1^2}{2}.\frac{\boldsymbol{L_1}}{\boldsymbol{D_1}} + \lambda_2 \frac{\rho.v_1^2}{2}.\frac{\boldsymbol{L_2}}{\boldsymbol{D_2}} + (k_1 + k_2)\frac{\rho v_1^2}{2} + k_3 \frac{\rho v_2^2}{2}$$

$$v_1 = \frac{4Q}{\pi . D_1^2} = \frac{4x0{,}1}{3{,}14x\ 0{,}15^2} = 5.66\ m/\boldsymbol{s}$$

$$v_2 = \frac{4Q}{\pi . D_2^2} = \frac{4x0{,}1}{3{,}14x\ 0{,}3^2} = 1.415\ m/\boldsymbol{s}$$

Let's Calculate $\boldsymbol{R_{e1}}$ **et** $\boldsymbol{R_{e2}}$ **:**

$$R_{e1} = \frac{V_1.D_1}{\nu} = \frac{5.66x0.15}{1.005x10^{-6}} = 8{,}44x10^5$$

$$R_{e2} = \frac{V_2.D_2}{\nu} = \frac{1.415x0.3}{1.005x10^{-6}} = 4{,}22x10^5$$

The correlation gives:

$$\frac{1}{\sqrt{\lambda_1}} = -1.8log\left(\frac{6.9}{R_{e1}} + \left(\frac{\varepsilon/D_1}{3.7}\right)^{1.11}\right) \Rightarrow \lambda_1 = 0.0182$$

$$\frac{1}{\sqrt{\lambda_2}} = -1.8log\left(\frac{6.9}{R_{e2}} + \left(\frac{\varepsilon/D_2}{3.7}\right)^{1.11}\right) \Rightarrow \lambda_2 = 0.0166$$

$\rho_{eau} g z_2 = \Delta H_{12}$

$$\Delta z = \Delta H' = 0.0186\frac{5.66^2}{2x9.81}.\frac{\mathbf{50}}{\mathbf{0.15}} + 0.0166\frac{1.415^2}{2x9.81}.\frac{\mathbf{160}}{\mathbf{0.30}} + (0{,}5+1)\frac{5.66^2}{2x9.81} + \frac{1.415^2}{2x9{,}81}$$

Solution of the exercise 9

Let's apply Bernoulli's equation between A and B :

$$P_A + \frac{1}{2}\rho_{eau}v^2{}_A + \rho_{eau} g z_A = P_B + \frac{1}{2}\rho_{eau}v^2{}_B + \rho_{eau} g z_B + \Delta H_{12}$$

We have : $z_1 = 0, z_2 = 15\ m$, $P_B = Patm$, $P_A = P_0$, $v_A = v_B = 0$ (Constant water level) , alors :

$$P_0 = P_{atm} + + \rho_{eau} g z_B + \Delta H_{12}$$

$$\Delta H = \lambda_1 \frac{\rho.v^2}{2}.\frac{\boldsymbol{L}}{\boldsymbol{d}} + +(k_1+k_2+k_3+k_4)\frac{\rho v^2}{2}$$

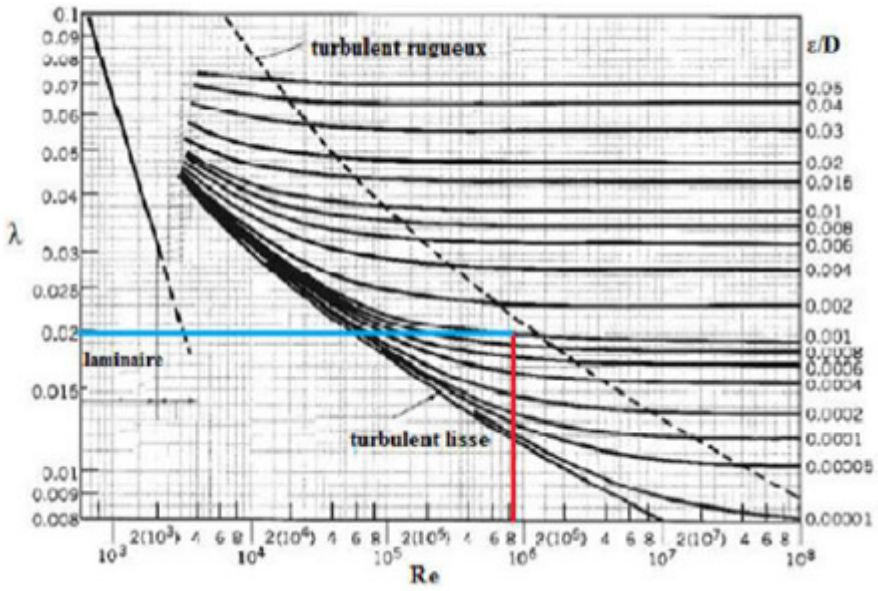
$$v = \frac{Q_v}{S} = \frac{4.Q_v}{\pi.d^2} = \frac{4x0.2}{\pi x0{,}3^2} = 2{,}83m/s$$

To calculate λ, it is necessary to calculate ε/d et Re :
The relative roughness ε/d =0,3/300 = 0,001

$R_{e1} = \frac{v.d}{\nu} = \frac{2{,}83x0.3}{1.005x10^{-6}} = 8{,}44776x10^5$, flow is turbulent

According to the Moody diagram, for ε/d = 0,001 and Re = 844776 , λ =0.0199

0,02 (voir la figure ci-dessous).



$$P_0 = P_{atm} + +\rho_{eau} g z_B + \lambda \frac{\rho . v^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}} + +(k_1 + k_2 + k_3 + k_4) \frac{\rho v^2}{2}$$

$$P_0 = 10^5 + 10^3 . 10.15 + 0.02 \frac{10^3 . 2{,}83^2}{2} . \frac{170}{0{,}3} + +(0{,}5 + 0{,}15 + 0{,}15 + 1) \frac{10^3 (2{,}83)^2}{2}$$

$$P_0 = 3\ 10^5$$

Solution of the exercise 10

Let's apply Bernoulli's equation between A and B:

$$P_1 + \frac{1}{2}\ \rho_{eau} {v^2}_1 + \rho_{eau} g z_1 = P_2 + \frac{1}{2}\ \rho_{eau} {v^2}_2 + \rho_{eau} g z_2 + \Delta H_{12}$$

We have : $z_1 = 80m, z_2 = 0\ m$, $P_1 = P_1 = Patm$, $v_1 = 0$:

$$\rho_{eau} g z_1 = \frac{1}{2}\ \rho_{eau} {v^2}_2 + \Delta H_{12}$$

$$\Delta H = \lambda \frac{\rho_{eau} . {v_2}^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}}$$

$$\rho_{eau} g z_B = \frac{1}{2}\ \rho_{eau} {v^2}_2 + \lambda . \frac{\rho_{eau} . {v_2}^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}}$$

$$\rho_{eau} g z_B = \frac{1}{2}\ \rho_{eau} {v^2}_2 + \lambda . \frac{\rho . {v_2}^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}} = \frac{1}{2}\ \rho_{eau} {v^2}_2 \left(1 + \lambda \frac{L}{d}\right)$$

$$v_2 = \sqrt{\frac{2 g z_B}{1 + \lambda \frac{L}{d}}} = \sqrt{\frac{2x9.81x80}{1 + 0.025 \frac{73}{0.05}}} = 6.47 m/s$$

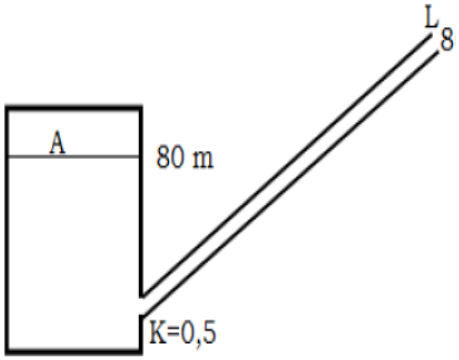
$$Q_v = v_2 . S_2 = v_2 . \pi . \frac{d^2}{4} = 6{,}47x3.14x \frac{0.05^2}{4} = 0{,}0127 m^3/s$$

Solution of the exercise 11

$$P_A + \frac{1}{2}\rho_h v^2{}_A + \rho_h g z_A = P_L + \frac{1}{2}\rho_h v^2{}_L + \rho_h g z_L + \Delta H_{12}$$

On a : $z_1 = 80\ m, z_2 = 86\ m$, $P_L = P_{atm}$, $P_L = P_{atm}$, $v_A = 0$ so :

$$P_A = P_{atm} + \frac{1}{2}\rho_h v^2{}_L + \rho_h g(z_L - z_L) + \Delta H_{12}$$



$$\Delta H = \lambda \frac{\rho_h . v_L{}^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}} + k \frac{\rho_h v_L{}^2}{2}$$

$$P_A = P_{atm} + \frac{1}{2}\rho_h v^2{}_L + \rho_h g(z_L - z_L) + \lambda \frac{\rho_h . v_L{}^2}{2} . \frac{\boldsymbol{L}}{\boldsymbol{d}} + k \frac{\rho_h v_L{}^2}{2}$$

$$v_L = \frac{Q_v}{S} = \frac{4.Q_v}{\pi.d^2} == 0{,}736 m/s$$

To calculate λ, it is necessary to calculate ε/d and Re :

The relative roughness ε/d =0,12/150 = 0,0008

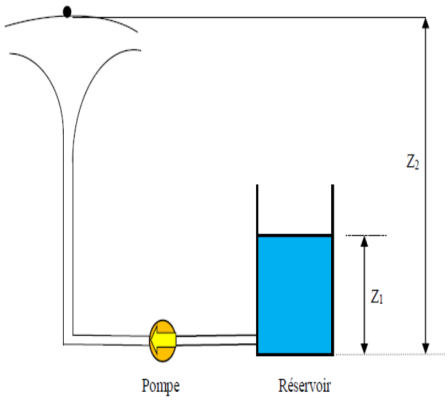
$R_{e1} = \frac{\rho v.d}{\eta} = \frac{v.d}{\nu} = \frac{0.736x0.15}{2{,}1x10^{-6}} = 5{,}6x10^4$, flow is turbulent

$$\frac{1}{\sqrt{\lambda}} = -1.8log\left(\frac{6.9}{R_{e2}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right) \Rightarrow \lambda = 0.023$$

so :

$$P_A = 10^5 + 840\text{x}9{,}81\text{x}6 + (1 + 0{,}023.\frac{\boldsymbol{150}}{\boldsymbol{0,15}} + 0{,}5)\ x\ \frac{840x0.736^2}{2}$$

Solution of the exercise 12



1- Calculate the speed:V

$$V = \frac{4q}{\pi . D^2} = \frac{4.2.10^{-3}}{\pi . 0.03^2} = 2.83 m/s$$

2- Calculate the Reynolds number: Re

$$R_{e1} = \frac{\rho v . d}{\eta} = \frac{2.83.0.03.10^3}{10^{-3}} = 84900$$

$2000 < Re < 10^5$ régime turbulent lisse.

3- Pressure drop coefficient: λ

Blasius's Formula: $\lambda = 0.316 . Re^{-1/4}$

$\lambda = 0.316 . (84900)^{-1/4} = 0.018$

The total pressure drop

Linear pressure drop:

$$J_L = \lambda \frac{\rho_h . v_L{}^2}{2} . \frac{L}{d} = 0{,}08 \frac{10^3 x15x(2{,}83)^2}{2.x0{,}03} = 160178 \text{ Pa}$$

Singular pressure drop:

$$J_s = K \frac{\rho . v^2}{2} = 0{,}3 \frac{10^3 . (2{,}83)^2}{2} = 1201\ Pa$$

4- Total power:

$$J_L + J_s = 160178 + 1200 = 161378 Pa$$

$$P_1 + \frac{1}{2}\rho v_2^2 + \rho_h g z_1 + \frac{P}{q_v} = P_2 + \frac{1}{2}\rho v_2^2 + \rho_h g z_L + \Delta H_{12}$$

$$(P_2 - P_1) + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho_h g(z_2 - z_1) + = \frac{P}{q_v} - \Delta H_{12}$$

$P_2 = P_1 = P_{atm}$, $v_2 = v_1$

$$\rho g(z_2 - z_1) + \rho_h g(z_2 - z_1) + = \frac{P}{q_v} - \Delta H_{12}$$

$$P = q_v . [\rho . g . (z_2 - z_1) + \Delta H_{12}]$$

$$P = 2.10^{-3}(10^3 x9{,}81(10 - 3) + 161378) =$$

460,096 w.

Power absorbed by the pump

$$P_a = \frac{P}{\tau} = \frac{460{,}096}{0{,}75} = 613{,}461 w$$

References

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